The radius of comparison of the crossed product by a finite group

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The comparison theory of projections is fundamental to the theory of von Neumann algebras, and is the basis for the type classification of factors. A C*-algebra might have few or no projections, in which case their comparison theory tells us little about the structure of the C*-algebra. The appropriate replacement for projections is positive elements. This idea was first introduced by Cuntz in [5] with a view to studying dimension functions on simple C*-algebras. Then the appropriate definition of the radius of comparison of C*-algebras was introduced by Andrew S. Toms in Section 6 of [11] to study exotic examples of simple amenable C*-algebras that are not $\mathcal{Z}$-stable. In the commutative setting, it is well known that the radius of comparison of $C(X)$ is dominated by one half of the covering dimension of X [4, 6]. Also, the comparison theory can be viewed as a non-commutative dimension theory [1]. See [4, 7, 8, 9, 10, 12] for the significant progress on the comparison theory.

In this talk, I will show that for an action $\alpha\colon G \to \mathrm{Aut}(A)$ of a finite group G on a unital simple stably finite C*-algebra A, the radii of comparison of A, the crossed product, and the fixed point algebra are related by

$$\mathrm{rc}(A^{\alpha}) \leq \mathrm{rc}(A) \qquad \text{and} \qquad \mathrm{rc}\big(C^*(G, A, \alpha)\big) \leq \frac{1}{\mathrm{card}\,(G)} \cdot \mathrm{rc}(A)$$

if α has the weak tracial Rokhlin property. Also, they are related by

$$\mathrm{rc}(A) \leq \mathrm{rc}\big(C^*(G, A, \alpha)\big) \leq \mathrm{rc}(A^{\alpha})$$

if α is tracially strictly approximately inner and $C^*(G, A, \alpha)$ is simple.

This talk is based on the speaker's Ph.D. dissertation under the supervision of N. Christopher Phillips and the results of [2]. I encourage the reader to go through the references displayed in this abstract, and check [3] for further details.

References:

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