

# *Book of Abstracts*

*2nd Zagreb Workshop on Operator Theory*

*ZWOT 2020*

*June 29–30, Zagreb*



# 1 General information

*2nd Zagreb Workshop on Operator Theory* will take place online on June 29 and 30, 2020 via Zoom. The workshop topics include (but are not limited to) Operator Algebras, Noncommutative Geometry and Preservers. 30 minute invited talks will take place real time via Zoom, while the contributed talks will be prerecorded and available online. There is no conference fee.

## Organizing and Scientific Committee

- Ljiljana Arambašić (University of Zagreb, Croatia)
- Tomislav Berić (University of Zagreb, Croatia)
- Matej Brešar (University of Ljubljana, Slovenia)
- Branimir Čačić (University of New Brunswick, Canada)
- Ilja Gogić (University of Zagreb, Croatia)

## 2 Invited Speakers

- Pere Ara (Universitat Autònoma de Barcelona, Spain)
- Francesca Arici (Leiden University, Netherlands)
- Joan Bosa (Universitat Autònoma de Barcelona, Spain)
- Matej Brešar (University of Ljubljana, Slovenia)
- Branimir Čačić (University of New Brunswick, Canada)
- Adam Dor-On (University of Copenhagen, Denmark)
- George Elliott (University of Toronto, Canada)
- Ilijas Farah (York University, Canada)
- Michael Frank (HTWK Leipzig, Germany)
- György Pál Gehér (University of Reading, UK)
- Alexander Guterman (Lomonosov Moscow State University, Russia)
- Matthew Kennedy (University of Waterloo, Canada)
- Bojan Kuzma (University of Primorska, Slovenia)
- Chi-Kwong Li (College of William and Mary, Williamsburg, USA)
- Martin Mathieu (Queen's University Belfast, UK)
- Lajos Molnár (University of Szeged, Hungary)
- Antonio M. Peralta (Universidad de Granada, Spain)
- Leonel Robert (University of Louisiana at Lafayette, USA)
- Peter Šemrl (University of Ljubljana, Slovenia)
- Aaron Tikuisis (University of Ottawa, Canada)

### 3 Schedule

Talk times are in the Central European Summer Time zone (CEST).

#### Monday, June 29

**Opening:** 09:55

**Chair:** L. Molnár

10:00-10:30 P. Šemrl: *Coexistence preservers*

10:40-11:10 M. Brešar: *Images of noncommutative polynomials*

11:20-11:50 A. Peralta: *Spinning the Gleason-Kahane-Żelazko and Kowalski-Słodkowski theorems*

12:00-12:30 B. Kuzma: *Approximate reducibility of unitary subgroups*

*Lunch break & discussion*

**Opening:** 14:55

**Chair:** P. Ara

15:00-15:30 C.-K. Li: *Joint numerical ranges and commutativity of matrices*

15:40-16:10 F. Arici: *Universal  $C^*$ -algebras from  $C^*$ -correspondences and the topology of circle bundles*

16:20-16:50 B. Čačić: *Principal bundles in noncommutative Riemannian geometry*

**Chair:** L. Robert

17:00-17:30 I. Farah: *Representations of simple  $C^*$ -algebras*

17:40-18:10 M. Kennedy: *Noncommutative Choquet simplices*

18:20-18:50 A. Tikuisis: *Classification of  $C^*$ -algebras and  $*$ -homomorphisms*

#### Tuesday, June 30

**Chair:** P. Šemrl

10:00-10:30 L. Molnár: *A new look at 2-local automorphisms*

10:40-11:10 A. Guterman: *Scrambling index and operators preserving its values*

11:20-11:50 G. P. Gehér: *An Uhlhorn-type generalisation of Wigner's unitary-antiunitary theorem*

12:00-12:30 M. Frank: *B-spline interpolation problem in Hilbert  $C^*$ -modules*

*Lunch break & discussion*

**Chair: G. Elliott**

15:00-15:30 P. Ara: *Sheaves and cohomology for  $C^*$ -algebras I*  
15:40-16:10 M. Mathieu: *Sheaves and cohomology for  $C^*$ -algebras II*  
16:20-16:50 A. Dor-On: *Doob equivalence and non-commutative peaking for Markov chains*

**Chair: A. Tikuisis**

17:00-17:30 J. Bosa: *The realization problem for von Neumann regular rings*  
17:40-18:10 L. Robert: *Lie ideals and normal subgroups in  $C^*$ -algebras*  
18:20-18:50 G. Elliott: *Thoughts on the classification problem for amenable  $C^*$ -algebras*

**Short version of the schedule**

<b>Monday, June 29</b>	<b>Tuesday, June 30</b>
<i>Opening: 09:55</i>	
Chair: L. Molnár	Chair: P. Šemrl
10:00-10:30 P. Šemrl 10:40-11:10 M. Brešar 11:20-11:50 A. Peralta 12:00-12:30 B. Kuzma	10:00-10:30 L. Molnár 10:40-11:10 A. Guterman 11:20-11:50 G.P. Gehér 12:00-12:30 M. Frank
<i>Lunch break &amp; discussion</i>	<i>Lunch break &amp; discussion</i>
<i>Opening: 14:55</i>	
Chair: P. Ara	Chair: G. Elliott
15:00-15:30 C.-K. Li 15:40-16:10 F. Arici 16:20-16:50 B. Ćaćić	15:00-15:30 P. Ara 15:40-16:10 M. Mathieu 16:20-16:50 A. Dor-On
Chair: L. Robert	Chair: A. Tikuisis
17:00-17:30 I. Farah 17:40-18:10 M. Kennedy 18:20-18:50 A. Tikuisis	17:00-17:30 J. Bosa 17:40-18:10 L. Robert 18:20-18:50 G. Elliott

## 4 Invited talks

### Sheaves and cohomology for $C^*$ -algebras I

Pere Ara

Universitat Autònoma de Barcelona

In these talks, we will introduce the concept of a sheaf of  $C^*$ -algebras, together with some basic examples. We discuss the categories of modules for which a cohomology theory will be constructed. These categories are typically non-abelian, and this makes more difficult to adapt the standard theory developed in Algebraic Geometry to this setting. We therefore briefly explain the concept of an exact category and how the cohomology groups are introduced using injective resolutions. One of the basic new ideas is the notion of a  $C^*$ -ringed space.

### Universal $C^*$ -algebras from $C^*$ -correspondences and the topology of circle bundles

Francesca Arici

Universiteit Leiden

In 1997 Pimsner described how to construct two universal  $C^*$ -algebras associated with an injective  $C^*$ -correspondence, now known as the Toeplitz–Pimsner and Cuntz–Pimsner algebras. In this talk I will recall their construction, focusing for simplicity on the case of a finitely generated projective correspondence. I will describe the associated six-term exact sequence in  $K(K)$ -theory and explain how these can be used in practice for computational purposes. Motivated by the theory of fibre bundles, I will describe how, in the case of a self-Morita equivalence, these exact sequences can be interpreted as an operator algebraic version of the classical Gysin sequence for circle bundles. If time allows, I will hint at how to extend this analogy to high sphere bundles.

## The realization problem for von Neumann regular rings

Joan Bosa  
Universitat Autònoma de Barcelona

The realization problem for von Neumann (vN) regular rings asks whether all conical refinement monoids arise from monoids induced by the projective modules over a vN regular ring. We will quickly overview this problem, and show the last developments on it.

References:

1. P. Ara, J. Bosa, E. Pardo, "The realization problem for finitely generated refinement monoids", *Selecta Mathematica* 26 (2020), no.3, 33.
2. P. Ara, J. Bosa, E. Pardo, A. Sims, "The Groupoids of Adaptable Separated graphs and Their Type semigroups." *IMRN*, 10.1093/imrn/rnaa022
3. P. Ara, J. Bosa E. Pardo "Refinement monoids and adaptable separated graphs," *Semigroup Forum*, 10.1007/s00233-019-10077-2

## Images of noncommutative polynomials

Matej Brešar  
University of Ljubljana and University of Maribor

Let  $F$  be a field, let  $f = f(X_1, \dots, X_m)$  be a noncommutative polynomial with coefficients in  $F$ , and let  $A$  be an  $F$ -algebra. We will discuss various questions concerning the *image of  $f$*  (in  $A$ ), which is defined to be the set  $f(A) = \{f(a_1, \dots, a_m) \mid a_1, \dots, a_m \in A\}$ . A special emphasis will be on the Waring type problem, asking about the existence of a positive integer  $N$  (independent of  $f$ , provided that  $f$  satisfies some natural restrictions) such that every element in  $A$  is a linear combination of  $N$  elements from  $f(A)$ . Our methods are algebraic and we are primarily interested in the case where  $A = M_n(F)$ , but the case where  $A = B(H)$  will also be considered.

## **Principal bundles in noncommutative Riemannian geometry**

Branimir Ćaćić  
University of New Brunswick

Noncommutative topological principal bundles, e.g., noncommutative circle bundles, are ubiquitous in theory of  $C^*$ -algebras. However, applications to mathematical physics, especially noncommutative generalisations of gauge theory, motivate a refinement of this notion to noncommutative Riemannian geometry in terms of spectral triples. In this talk, I'll sketch the resulting theory as it applies to noncommutative Riemannian circle bundles with orbits of constant length. This is based on joint work with Bram Mesland.

## **Doob equivalence and non-commutative peaking for Markov chains**

Adam Dor-On  
University of Copenhagen

The theory of Markov chains has applications in diverse areas of Mathematics and Physics such as group theory, dynamical systems, electrical networks and information theory. Connections with operator algebras however, seem to be mostly in the non-commutative scenario of quantum information.

In this talk we will show how questions about operator algebras of stochastic matrices, studied by Markiewicz and myself, motivate new problems in Markov chain theory. We will show how the study of harmonic functions allows for better classification results of our algebras, and how non-commutative peaking in the sense of Arveson can be completely characterized in terms of the stochastic matrix.

\*This is based on joint work with Xinxin Chen, Langwen Hui, Christopher Linden and Yifan Zhang, conducted as part of an undergraduate research project in Illinois Geometry Lab at UIUC.

## Thoughts on the classification problem for amenable C\*-algebras

George Elliott  
University of Toronto

Thirty years ago—thirty years after Glimm’s classification of uniformly hyperfinite (UHF) C\*-algebras—and fifty years after Gelfand and Naimark classified commutative C\*-algebras by their spectrum, an invariant that also is fundamental in the non-commutative case—, the prospect of classifying all (separable) amenable (= nuclear) C\*-algebras seemed perhaps to hover on the horizon.

Now, after thousands of pages of painstaking calculations, coupled with numerous conceptual advances, the restricted but robust Toms-Winter class of especially well-behaved simple separable amenable C\*-algebras (assumed to be Jiang-Su stable—not automatic but examples arising everywhere—and to satisfy the Universal Coefficient Theorem (UCT)—perhaps automatic) has been classified by means of a simple invariant (K-theoretic in nature), the possible values of which are also very simple.

(The classification is perhaps reminiscent of that of finite simple groups, although that is not formulated in terms of an invariant. One hopes that the C\*-algebra classification could also be fruitful.)

Given the additional information contained in more subtle invariants, such as the Cuntz semigroup, it does not seem unreasonable to continue to envisage the original goal.

## Representations of simple C\*-algebras

Ilijas Farah  
York University

In 1961 Glimm proved a deep dichotomy theorem on irreducible representations of separable, simple, C\*-algebras. Some parts of this theorem have been extended to not necessarily separable case by Sakai and others. I will discuss the possibility of extending full Glimm’s theorem to all C\*-algebras. This is a joint work with Daniel Calderón.



## ***B*-spline interpolation problem in Hilbert $C^*$ -modules**

Michael Frank

Hochschule für Technik, Wirtschaft und Kultur (HTWK) Leipzig, Fakultät  
Informatik und Medien, PF 301166, D-04251 Leipzig, Germany.

The *B*-spline interpolation problem corresponding to a  $C^*$ -valued sesquilinear form on a Hilbert  $C^*$ -module is introduced and its basic properties are studied. First, the problem is investigated in the case when the Hilbert  $C^*$ -module is self-dual. Extending a bounded  $C^*$ -valued sesquilinear form on a Hilbert  $C^*$ -module to a sesquilinear form on its second dual, some necessary and sufficient conditions for the *B*-spline interpolation problem to have a solution are provided.

Passing to the setting of Hilbert  $W^*$ -modules, the main result is presented by characterizing when the spline interpolation problem for the extended  $C^*$ -valued sesquilinear to the dual  $\mathcal{X}'$  of the Hilbert  $W^*$ -module  $\mathcal{X}$  has a solution. As a consequence, a sufficient condition is given characterizing orthogonally complemented submodules of a self-dual Hilbert  $W^*$ -module  $\mathcal{X}$  that are orthogonally complemented with respect to another  $C^*$ -inner product on  $\mathcal{X}$ . Finally, solutions of the *B*-spline interpolation problem for Hilbert  $C^*$ -modules over  $C^*$ -ideals of  $W^*$ -algebras are extensively discussed. Several examples are provided to illustrate the existence or lack of a solution for the problem.

This is joint work with R. Eskandari (Farhangian University, Tehran), V. M. Manuilov (Moscow State University, Moscow), M. S. Moslehian (Ferdowsi University of Mashhad, Mashhad).

## **An Uhlhorn-type generalisation of Wigner's unitary-antiunitary theorem**

György Pál Gehér

University of Reading, UK

Let  $H$  be a Hilbert space and  $P(H)$  be the projective space of all quantum pure states. Wigner's theorem states that every bijection  $\phi: P(H) \rightarrow P(H)$  that preserves the quantum angle between pure states is automatically induced by either a unitary or an antiunitary operator  $U: H \rightarrow H$ . Uhlhorn's

theorem generalises this result for bijective maps  $\phi$  that are only assumed to preserve the quantum angle  $\frac{\pi}{2}$  (orthogonality) in both directions. In this talk we consider the corresponding structural problem for bijections that preserve only one fixed quantum angle  $\alpha$  in both directions,  $0 < \alpha < \frac{\pi}{2}$ .

*Partly joined work with Michiya Mori (University of Tokyo)*

References:

1. G.P. Gehér, Symmetries of Projective Spaces and Spheres, *Int. Math. Res. Not. IMRN* (2020), 2205–2240.
2. G.P. Gehér, and M. Mori, The structure of maps on the space of all quantum pure states that preserve a fixed quantum angle, *submitted*
3. C.-K. Li, L. Plevnik, and P. Šemrl, Preservers of matrix pairs with a fixed inner product value, *Oper. Matrices* **6** (2012), 433–464.

## Scrambling index and operators preserving its values

A.E. Guterman

Lomonosov Moscow State

This talk is based on the joint work with A.M. Makshev.

Scrambling index is a fundamental matrix invariant. Shortly, a matrix is said to be scrambling if for any two rows there is a column having positive intersections with both these rows. A scrambling index of a non-negative matrix  $A$  is a minimal  $k$  such that  $A^k$  is a scrambling matrix, and 0, if such  $k$  does not exist. Correspondingly, a scrambling index of a primitive directed graph  $G$  is the smallest positive integer  $k = k(G)$  such that for any pair of vertices  $u, v$  of  $G$  there exists a vertex  $w$  of  $G$  such that there are directed walks of length  $k$  from  $u$  to  $w$  and from  $v$  to  $w$ . If  $G$  is not primitive, then it can appear that the integer  $k$  described above does not exist. In this case we say that  $k(G) = 0$ , otherwise we define  $k(G)$  as in the primitive case.

The scrambling index is important in several applications. In particular, if  $A$  is an  $n \times n$  non-negative primitive stochastic matrix with a non-unit eigenvalue  $\lambda$ , and  $k$  is the scrambling index of  $G(A)$ , then  $|\lambda| \leq (\tau_1(A^k))^{1/k} < 1$ , where  $\tau_1$  is a certain matrix invariant, usually called Dobrushin coefficient.

Also scrambling index provides lower bounds for the length of reset words for synchronizing automata, since it gives a lower bound for the exponent of the matrix representing this automata.

More applications are in the theory of memoryless communication systems and related areas. Scrambling index for primitive graphs was an object of intensive investigations starting from the works by Seneta, Paz, Akelbek, Kirkland, and others.

We prove that for non-primitive digraphs on  $n$  vertices the following bound  $k(G) \leq 1 + \left\lceil \frac{(n-2)^2+1}{2} \right\rceil$  is true. We characterize the graphs with the maximal scrambling index and characterize non-primitive graphs possessing positive scrambling index.

In addition we investigate linear transformations preserving scrambling index. In particular, we show that these maps are always bijective. The structure of linear maps preserving only several values of the scrambling index is also characterized.

## Noncommutative Choquet simplices

Matthew Kennedy  
University of Waterloo

Recently, Ken Davidson and I introduced a new framework for noncommutative convexity, along with a corresponding noncommutative Choquet theory that generalizes much of classical Choquet theory. In this talk, I will discuss a notion of noncommutative Choquet simplex, which generalizes the classical notion of Choquet simplex and turns out to play an analogous role in noncommutative dynamics. I will discuss some applications, including the following extension of Glasner and Weiss's characterization of groups with Kazhdan's property (T): a group has property (T) if and only if whenever it acts on a  $C^*$ -algebra, the set of invariant states is affinely homeomorphic to the state space of a  $C^*$ -algebra. This is joint work with Eli Shamovich.

## Approximate reducibility of unitary subgroups

Bojan Kuzma

University of Primorska, and IMFM, and Moscow Center for Fundamental  
and Applied Mathematics

A group of 2-by-2 block matrices whose each member is block-upper triangular is always reducible. For subgroups of a unitary group it is shown that the same is true if block upper-triangularity is replaced by a weaker condition that, for every element of a given unitary subgroup, the norm of a block at position  $(2, 1)$  is sufficiently small.

This is a joint work with M. Mastnak, M. Omladič, and H. Radjavi.

## Joint numerical ranges and commutativity of matrices

Chi-Kwong Li

College of William and Mary

The connection between the commutativity of a family of  $n \times n$  matrices and the generalized joint numerical ranges is studied. For instance, it is shown that  $\mathcal{F}$  is a family of mutually commuting normal matrices if and only if the joint numerical range  $W_k(A_1, \dots, A_m)$  is a polyhedral set for some  $k$  satisfying  $|n/2 - k| \leq 1$ , where  $\{A_1, \dots, A_m\}$  is a basis for the linear span of the family; equivalently,  $W_k(X, Y)$  is polyhedral for any two  $X, Y \in \mathcal{F}$ . More generally, characterization is given for the  $c$ -numerical range  $W_c(A_1, \dots, A_m)$  to be polyhedral for any  $n \times n$  matrices  $A_1, \dots, A_m$ . Other results connecting the geometrical properties of the joint numerical ranges and the algebraic properties of the matrices are obtained. Implications of the results to representation theory, and quantum information science are discussed.

This is based on a joint paper with Yiu-Tung Poon and Yashu Wang.

## Sheaves and cohomology for C\*-algebras II

Martin Mathieu  
Queen's University Belfast

In these talks, we will introduce the concept of a sheaf of C\*-algebras, together with some basic examples. We discuss the categories of modules for which a cohomology theory will be constructed. These categories are typically non-abelian, and this makes more difficult to adapt the standard theory developed in Algebraic Geometry to this setting. We therefore briefly explain the concept of an exact category and how the cohomology groups are introduced using injective resolutions. One of the basic new ideas is the notion of a C\*-ringed space.

## A new look at 2-local automorphisms

Lajos Molnár  
University of Szeged, and Budapest University of Technology and  
Economics

By a well-known result of P. Šemrl, every 2-local automorphism of the full operator algebra over a separable Hilbert space is necessarily an automorphism. This means that every map of that algebra which, on each two-point subset, equals an algebra automorphism (depending on the two-point sets in question) is necessarily itself an algebra automorphism. This can be formulated as saying that the automorphism group of the algebra is 2-reflexive. In this talk we present recent results in which we strengthen that result quite substantially for \*-automorphisms. We show that one can compress the defining two equations of 2-local \*-automorphisms into one single equation, hence weakening the requirement significantly, but still keeping essentially the conclusion that such maps are necessarily \*-automorphisms. The observation leads also to the result that the full isometry group (not only the subgroup of linear isometries) of the full operator algebra is 2-reflexive.

## **Spinning the Gleason-Kahane-Żelazko and Kowalski-Słodkowski theorems**

Antonio M. Peralta

Departamento de Análisis Matemático, Universidad de Granada

We shall present two recent spherical variants of the Gleason-Kahane-Żelazko and Kowalski-Słodkowski theorems with different applications to the study of weak-2-local isometries on uniform algebras and Lipschitz algebras. Among the applications, we present some positive solutions to problems posed by O. Hatori, T. Miura, H. Oka and H. Takagi and by L. Molnár on 2-local isometries.

## **Lie ideals and normal subgroups in $C^*$ -algebras**

Leonel Robert

University of Louisiana at Lafayette

I will present various results on Lie ideals of  $C^*$ -algebras. I will then discuss how these results can be combined with properties of the exponential map to obtain similar results on normal subgroups of the groups of invertibles and unitaries of a  $C^*$ -algebra. I will pose some open questions along the way. I will also comment on the role played by "regularity properties"—of the kind encountered in the classification of simple nuclear  $C^*$ -algebras—in these questions.

## **Coexistence preservers**

Peter Šemrl

University of Ljubljana

We will describe the general form of coexistence preservers on effect algebras. The talk is based on two papers, the first one with György Gehér, and the second one with Michiya Mori.

## Classification of $C^*$ -algebras and $*$ -homomorphisms

Aaron Tikuisis  
University of Ottawa

Over a long time, there has been an active program to classify nice (simple separable nuclear)  $C^*$ -algebras using K-theoretic invariants; that is to say, to show that two such  $C^*$ -algebras are isomorphic whenever their invariants are the same. I will discuss some history and developments around this problem, including a mention of joint work with Carrion, Gabe, Schafhauser, and White, where we give a new proof of a key case via an abstract classification of  $*$ -homomorphisms.

## 5 Contributed talks

### Diagonal ASH Algebras with stable rank one

Mihai Alboiu  
University of Toronto

In this talk, we introduce a subclass of recursive subhomogeneous algebras, in which each of the pullback maps is diagonal in a suitable sense. We define the notion of a diagonal map between two such algebras and show that every simple inductive limit of these algebras with diagonal maps has stable rank one. As an application, we prove that for any infinite compact metric space  $T$  and minimal homeomorphism  $h: T \rightarrow T$ , the associated dynamical crossed product  $C^*(\mathbb{Z}, T, h)$  has stable rank one. This affirms a conjecture of Archey, Niu, and Phillips. We also verify that the Toms-Winter Conjecture holds for minimal crossed products of this type. This talk is based on joint work with James Lutley.

### Symmetrized strong Birkhoff–James orthogonality in $C^*$ -algebras

Ljiljana Arambašić  
University of Zagreb

In this talk we present some results on mutual strong Birkhoff–James orthogonality in two classical  $C^*$ -algebras: the  $C^*$ -algebra  $\mathbb{B}(H)$  of all bounded linear operators on a complex Hilbert space  $H$  and a commutative, possibly nonunital,  $C^*$ -algebra. With the help of the induced graph it is shown that this relation alone can characterize right invertible elements. Moreover, in the case of commutative unital  $C^*$ -algebras, it can detect the existence of a point with a countable local basis in the corresponding compact Hausdorff space.

This is a joint work with A. Guterman, B. Kuzma, R. Rajić and S. Zhilina.



## The radius of comparison of the crossed product by a finite group

M. Ali Asadi-Vasfi  
University of Tehran

The comparison theory of projections is fundamental to the theory of von Neumann algebras, and is the basis for the type classification of factors. A  $C^*$ -algebra might have few or no projections, in which case their comparison theory tells us little about the structure of the  $C^*$ -algebra. The appropriate replacement for projections is positive elements. This idea was first introduced by Cuntz in [5] with a view to studying dimension functions on simple  $C^*$ -algebras. Then the appropriate definition of the radius of comparison of  $C^*$ -algebras was introduced by Andrew S. Toms in Section 6 of [11] to study exotic examples of simple amenable  $C^*$ -algebras that are not  $\mathcal{Z}$ -stable. In the commutative setting, it is well known that the radius of comparison of  $C(X)$  is dominated by one half of the covering dimension of  $X$  [4, 6]. Also, the comparison theory can be viewed as a non-commutative dimension theory [1]. See [4, 7, 8, 9, 10, 12] for the significant progress on the comparison theory.

In this talk, I will show that for an action  $\alpha: G \rightarrow \text{Aut}(A)$  of a finite group  $G$  on a unital simple stably finite  $C^*$ -algebra  $A$ , the radii of comparison of  $A$ , the crossed product, and the fixed point algebra are related by

$$\text{rc}(A^\alpha) \leq \text{rc}(A) \quad \text{and} \quad \text{rc}(C^*(G, A, \alpha)) \leq \frac{1}{\text{card}(G)} \cdot \text{rc}(A)$$

if  $\alpha$  has the weak tracial Rokhlin property. Also, they are related by

$$\text{rc}(A) \leq \text{rc}(C^*(G, A, \alpha)) \leq \text{rc}(A^\alpha)$$

if  $\alpha$  is tracially strictly approximately inner and  $C^*(G, A, \alpha)$  is simple.

This talk is based on the speaker's Ph.D. dissertation under the supervision of N. Christopher Phillips and the results of [2]. I encourage the reader to go through the references displayed in this abstract, and check [3] for further details.

References:

- [1] M. B. Asadi and M. A. Asadi-Vasfi, *The radius of comparison of the tensor product of a  $C^*$ -algebra with  $C(X)$* , preprint (arXiv:2004.03013v1 [math.OA]).

- [2] M. A. Asadi-Vasfi, *The radius of comparison of the crossed product by a tracially strictly approximately inner action*, in preparation.
- [3] M. A. Asadi-Vasfi, N. Golestani, and N. C. Phillips, *The Cuntz semi-group and the radius of comparison of the crossed product by a finite group*, preprint (arXiv: 1908.06343v1 [math.OA]).
- [4] B. Blackadar, L. Robert, A. P. Tikuisis, A. S. Toms, W. Winter, *An algebraic approach to the radius of comparison*, Trans. Amer. Math. Soc. **364** (2012), 3657–3674. (2012).
- [5] J. Cuntz, *Dimension functions on simple  $C^*$ -algebras*, Math. Ann. **233** (1978), 145–153.
- [6] G. A. Elliott and Z. Niu, *On the radius of comparison of a commutative  $C^*$ -algebra*, Canad. Math. Bull. **56**(2013), 737–744.
- [7] Z. Niu, *Mean dimension and AH-algebras with diagonal maps*, J. Funct. Anal. **266**(2014), 4938–4994.
- [8] I. Hirshberg and N. C. Phillips, *A simple nuclear  $C^*$ -algebra with an internal asymmetry*, preprint (arXiv: 1909.10728v1 [math.OA]).
- [9] N. C. Phillips, *The  $C^*$ -algebra of a minimal homeomorphism with finite mean dimension has finite radius of comparison*, preprint (arXiv: 1605.07976v1 [math.OA]).
- [10] N. C. Phillips, *Large subalgebras*, preprint (arXiv: 1408.5546v2 [math.OA]).
- [11] A. S. Toms, *Flat dimension growth for  $C^*$ -algebras*, J. Funct. Anal. **238**(2006), 678–708.
- [12] A. S. Toms and W. Winter, *The Elliott conjecture for Villadsen algebras of the first type*, J. Funct. Anal. **256**(2009), 1311–1340.

## Sequence dominance in shift-invariant spaces

Tomislav Berić  
University of Zagreb

For a given function  $\psi \in L^2(\mathbb{R})$  we study the system of integer translates  $B_\psi = \{T_k\psi : k \in \mathbb{Z}\}$ , where  $T_k$  is the translation operator. Numerous properties of  $B_\psi$  can be described via its periodization function  $p_\psi(\xi) = \sum_{k \in \mathbb{Z}} \left| \widehat{\psi}(\xi + k) \right|^2$ . For  $\psi$  we define its associated coefficient space  $\text{Cof}_\psi$  as the set of all the sequences  $(c_k)_{k \in \mathbb{Z}}$  for which  $\sum c_k T_k \psi$  converges in the  $L^2$  norm (with respect to the ordering  $0, 1, -1, 2, -2, \dots$  of  $\mathbb{Z}$ ). There are two important special cases: when  $\text{Cof}_\psi$  contains  $\ell^2(\mathbb{Z})$ , in which case we say that  $B_\psi$  has the (H)–property, and when  $\text{Cof}_\psi$  is contained in  $\ell^2(\mathbb{Z})$ , when we say that  $B_\psi$  has the (B)–property. Characterization of the (H)–property via the periodization function is already known. The (B)–property seems to be much more difficult to characterize and we will characterize it in two important special cases: when the periodization function has a certain degree of smoothness and when the system  $B_\psi$  has the (H)–property alongside the (B)–property.

This is joint work with Hrvoje Šikić.

## Bounded normal generation for commutator subgroups of unitary groups of $C^*$ -Algebras

Abhinav Chand  
University of Louisiana at Lafayette

Essential simplicity, i.e. simplicity modulo the center, of the commutator subgroups of unitary and invertible elements of a  $C^*$ -algebra has been investigated by various authors, starting with Kadison’s paper: ”Infinite Unitary groups”. In 2019, Leonel Robert answered these simplicity questions using the concept of Lie ideal and results that go back to Herstein. Among other things, Robert showed that if  $A$  is a simple unital  $C^*$ -algebra containing a non-trivial projection, then the commutator subgroup  $(U_0(A), U_0(A))$  is essentially simple. In this talk we discuss how Robert’s result can be improved on to obtain ”bounded normal generation” for  $(U_0(A), U_0(A))$ , assuming

that  $A$  has a non-trivial projection and further regularity properties on  $A$ . A group has the bounded normal generation property if for every group element  $g \neq 1$  we can find a natural number  $n$  such that every element can be written as a word of length at most  $n$  using elements of the conjugacy classes of  $g$  and  $g^{-1}$ . We will give a sketch of the proof for  $A$  a purely infinite  $C^*$ -algebra in which case  $(U_0(A), U_0(A)) = U_0(A)$ .

This is joint work with Leonel Robert (University of Louisiana at Lafayette)

### **Asymptotic dimension and coarse embeddings in the quantum setting**

Alejandro Chávez-Domínguez  
University of Oklahoma

We generalize the notions of asymptotic dimension and coarse embeddings from metric spaces to quantum metric spaces in the sense of Kuperberg and Weaver. We show that quantum asymptotic dimension behaves well with respect to metric quotients and direct sums, and is preserved under quantum coarse embeddings. Moreover, we prove that a quantum metric space that equi-coarsely contains a sequence of expanders must have infinite asymptotic dimension. This is done by proving a quantum version of a vertex-isoperimetric inequality for expanders, based upon a previously known edge-isoperimetric one due to Temme, Kastoryano, Ruskai, Wolf, and Verstraete. Joint work with Andrew Swift.

### **Positive solutions of the operator equation $AXB = C$**

Dragana Cvetković Ilić  
Faculty of Science and Mathematics, University of Niš

We will discuss the existence of a positive solution of the equation  $AXB = C$ . This problem was considered in different settings but only under additional conditions including that of regularity, as well as under certain range

conditions such as  $\mathbb{R}(B) \subseteq \overline{\mathbb{R}(A^*)}$ . We will answer this question of the existence of a positive solution of the operator equation  $AXB = C$  without any additional range or regularity assumptions using two well-known results of Douglas and Zoltán. Also we will give a general form of a positive solution and consider some possible applications.

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## Polar decomposition of the linear pencil

Slaviša Djordjević

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For a pair of operators  $A, B \in \mathcal{B}(H)$ , the operator  $A - \lambda B$ ,  $\lambda \in \mathbb{C}$ , is called the *linear pencil* of an ordered pair  $(A, B)$ . The *spectrum* of a linear pencil is the set

$$\sigma(A, B) = \{\lambda \in \mathbb{C} : A - \lambda B \text{ is not invertible}\},$$

and  $\rho(A, B) = \mathbb{C} \setminus \sigma(A, B)$  is called the *resolvent set* of the linear pencil  $(A, B)$ .

We recall that the polar decomposition of any bounded linear operator  $T$  between complex Hilbert spaces is a canonical factorization as the product of a partial isometry and a non-negative operator, that is,  $T = U|T|$ , where  $U$  is a partial isometry and  $|T| = \sqrt{T^*T}$  is a non-negative operator.

In this talk we introduce *the (spherical) polar decomposition* of a linear pencil of an ordered pair  $(A, B)$  and we give some properties of it. Using this way of decomposition of a pair of operators, we are able to define several type of Aluthge transforms for a linear pencil of operators.

Co-authors: Jasang Yoon and Jaewoong Kim

## Centrally stable algebras

Ilja Gogić

University of Zagreb

Motivated by Vesterstrøm's characterization of unital weakly central  $C^*$ -algebras via the centre-quotient property, we introduce the class of *centrally stable algebras*, as algebras  $A$  with the property that for any algebra epimorphism  $\phi : A \rightarrow B$ , the centre  $Z(B)$  of  $B$  is equal to  $\phi(Z(A))$ , the image of the centre of  $A$ .

After providing some examples and basic observations, we establish our main result which states that a finite-dimensional unital algebra  $A$  over a perfect field  $\mathbb{F}$  is centrally stable if and only if and only if

$$A \cong (C_1 \otimes_{\mathbb{F}_1} A_1) \times \cdots \times (C_r \otimes_{\mathbb{F}_r} A_r),$$

where each  $\mathbb{F}_i$  is a finite field extension of  $\mathbb{F}$ ,  $C_i$  is a commutative  $\mathbb{F}_i$ -algebra, and  $A_i$  is a central simple  $\mathbb{F}_i$ -algebra.

This is joint work with Matej Brešar (University of Ljubljana and University of Maribor).

### **Characterisation of flow equivalence relation for positive matrices over some unital rings**

Pavle Goldstein  
University of Zagreb

We give a Franks' - type characterisation of flow-equivalence relation for positive matrices over some unital rings (other than  $\mathbb{Z}$ ). We discuss the consequences of this result for classification of certain Cuntz-Krieger-Pimsner algebras.

Joint work with I. Ban.

### **The covariant Stone-von Neumann uniqueness theorem**

Lara Ismert  
Embry-Riddle Aeronautical University

In this talk, we formulate and prove a version of the Stone-von Neumann Theorem for every  $C^*$ -dynamical system of the form  $(G, \mathbb{K}(\mathcal{H}), \alpha)$ , where  $G$  is a locally compact Hausdorff abelian group and  $\mathcal{H}$  is a Hilbert space. The novelty of our work stems from our representation of the Weyl Commutation Relation on Hilbert  $\mathbb{K}(\mathcal{H})$ -modules instead of just Hilbert spaces, and our introduction of two additional commutation relations, which are necessary to obtain a uniqueness theorem.

## On fundamental groups of certain property (T) factors

Krishnendu Khan  
Vanderbilt University

Calculation of Fundamental groups of type  $\text{II}_1$  factors is, in general, an extremely important and difficult problem. In this direction, a conjecture due to A. Connes states that the Fundamental group of the group von Neumann algebra  $\mathcal{L}(G)$ , where  $G$  is an i.c.c. property (T) group, is trivial.

In this talk, I shall provide the first examples of property (T) group factors with trivial Fundamental group. This talk is based on a recent joint work with Ionut Chifan, Sayan Das and Cyril Houdayer.

## Improving estimates for discrete polynomial averaging operators

Vjekoslav Kovač  
University of Zagreb

It is a well-known property of all averaging-type operators that their operator norms are at most 1, simply as a consequence of the triangle inequality. In general, constant 1 cannot be improved, even when one considers various  $L^p \rightarrow L^q$  estimates for  $p \neq q$ . However, certain averaging operators allow a significant improvement of this constant, which finds applications to various problems in the harmonic analysis. Here we study averaging operators in a discrete polynomial setting, and prove sharp improving  $\ell^p \rightarrow \ell^q$  estimates in a close-to-optimal range of exponents  $(p, q)$ . This is joint work with R. Han, M. T. Lacey, F. Yang (Georgia Institute of Technology), and J. Madrid (University of California, Los Angeles).



## Internal categories of factors

Juan Orendain

Universidade Federal de Santa Catarina/UNAM

A natural extension of the Haagerup standard form construction to finite index inclusions of factors, and more generally to finite morphisms between semisimple von Neumann algebras, was proposed by Bartels, Douglas and Hénriques in their quest for a symmetric monoidal tricategory of coordinate-free conformal nets. This extension is shown to form a tensor functor, which together with a corresponding functorial extension of Connes fusion, provides factors, finite index morphisms, bimodules and equivariant intertwiners with the structure of a category internal to symmetric monoidal categories. The problem of existence of an internal category of factors, not-necessarily finite index morphisms, bimodules and equivariant intertwiners remains open.

We explain how this problem relates to classical constructions and arguments in non-abelian algebraic topology and how to use minimizing conditions and general constructions in the theory of double categories to provide formal solutions to this and related problems.

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## **On (strong) Birkhoff–James orthogonality in Hilbert $C^*$ -modules**

Rajna Rajić  
University of Zagreb

In this talk, we present some results on the Birkhoff–James orthogonality and its strong version in Hilbert  $C^*$ -modules. We describe the class of full Hilbert  $C^*$ -modules in which the (strong) Birkhoff–James orthogonality is a symmetric relation. We also characterize the class of (surjective) linear mappings  $\Phi : \mathbb{B}(H) \rightarrow \mathbb{B}(H)$  that preserve the (strong) Birkhoff–James orthogonality. This research was supported in part by the Croatian Science Foundation under the project IP-2016-06-1046.

## **Positive operators as real valued functions**

Andriamanankasina Ramanantoanina  
Central European University, Hungary

This talk is a short report of my recent work with Lajos Molnár. We explored some functional representations of positive Hilbert space operators. These are embeddings of the set of positive operators into the set of non-negative real valued functions associated to positive operators. The set of positive operators carries two important order structures: the usual order or Löwner order and the spectral order or Olson order, and the embeddings we are studying transport these orders on operators to pointwise order on functions. Moreover, they present some very interesting algebraic and topological properties.

## Convergence and $L^p$ estimate of generalized ergodic averages

Mario Stipčić  
University of Zagreb

We discuss paraproduct-type bilinear operators that generalize ergodic averages as well as martingales. We establish their  $L^p$  convergence for a certain range of exponents while discussing  $L^p$  estimate of each bilinear operator and pose an open question about their a.e. convergence.

## Formulas for inversion of frame multipliers

Diana Stoeva  
Austrian Academy of Sciences, Acoustics Research Institute

Frames for Hilbert spaces extend orthonormal bases allowing redundancy and still provide perfect and stable reconstruction. This makes them very useful for many real-life applications, e.g. in signal and image processing. In signal processing, frame multipliers can be seen as a particular way to implement time-variant filters. Theoretically, they can be described as operators which combine analysis of a signal via a frame, modification of the obtained coefficients via a scalar sequence, and synthesis via a (possibly different) frame leading to a modified signal. They can also be seen as a generalization of frame operators, but in contrast to frame operators, which are always invertible, multipliers may fail the invertibility property. In this talk, first we briefly recall the concepts of frame and frame multiplier. Then we give an overview on the investigation of inversion of multipliers during the last years, focusing on the following points: sufficient conditions for invertibility; formulas for the inverses aiming efficient computations; representation of the inverses as multipliers with specific form, motivated by the formula valid in the case of non-redundant frames. References are given for further details on the topics. The talk is based on a joint work with Peter Balazs.

## Classification of $C^*$ -algebras and the invariants

Kun Wang  
University of Puerto Rico

In my talk, I will talk about classification of AH algebras with the ideal property, with no dimension growth, and have finitely many ideals. In particular, we will introduce a invariant including Hausdorffified algebraic  $K_1$  group and another refined version of the invariant. At last, we show that the Hausdorffified algebraic  $K_1$  part is not necessary for our setting.

## 6 Participants

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18. Abhinav Chand (University of Louisiana at Lafayette)
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28. Emilie Elkiaer (ENS Lyon)
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32. Michael Frank (HTWK Leipzig)
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