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Let S be a compact Hausdorff, extremally disconnected space with no isolated points. Let G be a countably infinite group of homeomorphisms of S. Let the action of G on S have a free dense orbit. Then we prove that, on a generic subset of S, the orbit equivalence relation coming from this action can also be obtained by an action of the Dyadic Group, $\bigoplus \mathbb{Z}_2$.As an application, we show that if M is the monotone cross product C^*-algebra, arising from the natural action of G on $C(S)$, and if the projection lattice in $C(S)$ is countably generated then M can be approximated by an increasing sequence of finite dimensional subalgebras. On each S, in a large family, we construct an action of $\bigoplus \mathbb{Z}_2$ with a free, dense orbit. Using this we exhibit a huge family of small monotone complete C^*-algebras, $(B_\lambda, \lambda \in \Lambda)$ with the following properties: (i) Each B_λ is a Type III factor which is not a von Neumann algebra. (ii) Each B_λ is a quotient of the Pedersen-Borel envelope of the Fermion algebra and hence is strongly hyperfinite. The cardinality of Λ is 2^c, where $c = 2^{\aleph_0}$.When $\lambda \neq \mu$ then B_λ and B_μ take different values in the classification semi-group; in particular, they cannot be isomorphic. (Joint work with Kazuyuki Saitô.)