THE BEST APPROXIMATION PROBLEM IN HILBERT C^*-MODULES

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Abstract. Let $V \subseteq B(H,K)$ be a Hilbert C^*-module over a C^*-algebra $\mathcal{A} \subseteq B(H)$, and $X, Y \in V$. We study a problem of finding $A \in B(H)$ such that $|X + YA| \leq |X + YB|$ for all $B \in \mathcal{A}$. We give necessary and sufficient conditions for the existence and uniqueness of a solution, and analyze conditions under which one can find a solution in the double commutant $\mathcal{A}''$. We provide a description of all Hilbert C^*-modules V for which there is a solution to the above problem for all $X, Y \in V$.

This is a joint work with Ljiljana Arambašić.