

# OPERATOR VARIANCE-COVARIANCE INEQUALITY

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ABSTRACT. Let  $(\mathcal{X}, \langle \cdot, \cdot \rangle)$  be a semi-inner product module over a  $C^*$ -algebra  $\mathcal{A}$ . For arbitrary  $n \in \mathbb{N}$  and  $x_1, \dots, x_n \in \mathcal{X}$  we study the so-called  $n \times n$  Gram matrix  $[\langle x_i, x_j \rangle]$  with entries in  $\mathcal{A}$ , construct a non-decreasing sequence of positive matrices in  $M_n(\mathcal{A})$  which is majorized by  $[\langle x_i, x_j \rangle]$  and apply it to obtain generalizations of covariance inequality, an extension of the Ostrowski inequality and an improvement of the Kantorovich inequality involving operator means.

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