Pro-Banach dynamical systems with C*-crossed products

Maria Joița

Department of Physics and Applied Mathematics, University of Bucharest, Bd. Regina Elisabeta nr. 4-12, Bucharest, Romania
[mjoita@fmi.unibuc.ro]

Pro-Banach *-algebras (pro-C*-algebras) are generalizations of Banach *-algebras (C*-algebras). The topology on a pro-Banach *-algebra (pro-C*-algebra) is determined by a directed family of submultiplicative *-seminorms (C*-seminorms). The class of pro-C*-algebras is bigger than the class of C*-algebras, for example, $C_{cc}[0,1]$, the *-algebra of all continuous complex valued functions on $[0,1]$ with the topology "cc" of uniform convergence on the countable compact subsets of $[0,1]$, is a pro-C*-algebra which is not isomorphic to any C*-algebra. By analogy with the case of Banach *-algebras with bounded approximate unit, A. Inoue (19971) constructed the enveloping pro-C*-algebra of a pro-Banach *-algebra with bounded approximate unit. S. J. Bhatt and D. J. Karia [Topological algebras with C*-enveloping algebras, Proc. Indian Acad. Sci (Math. Soc.) 102 (1993), 201-215] proved sufficient and necessary conditions under which a pro-Banach *-algebra with bounded approximate unit admits a C*-algebra as enveloping pro-C*-algebra.

A pro-Banach dynamical system is a triple $(G; A; \alpha)$, where $G$ is a locally compact group, $A$ is a pro-Banach *-algebra with approximate unit and $\alpha$ is a continuous action of $G$ on $A$ (this is, the map $g \rightarrow \alpha_g$ from $G$ to Aut($A$) is a group morphism and the map $g \rightarrow \alpha_g(a)$ from $G$ to $A$ is continuous for each $a \in A$). If the action $\alpha$ is $G$-invariant (this is, there is a cofinal subset of $G$-invariant continuous submultiplicative *-seminorms, $p(\alpha_g(a)) \leq M_p p(a)$ for some $M_p > 0$ and for all $a \in A$ and $g \in G$), using the same techniques as in the case of the construction of the crossed product associated to a C*-dynamical system, we associate a pro-C*-algebra to a pro-Banach dynamical system, called pro-C*-crossed product. By definition, the pro-C*-crossed product associated to a pro-Banach dynamical system $(G, A, \alpha)$ is the enveloping pro-C*-algebra associated to the covariance algebra $L^1(G, A, \alpha)$.

In this talk, we will discuss about pro-Banach dynamical systems with C*-crossed products.