

Pro-Banach dynamical systems with C^* -crossed products

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Pro-Banach $*$ -algebras (pro- C^* -algebras) are generalizations of Banach $*$ -algebras (C^* -algebras). The topology on a pro-Banach $*$ -algebra (pro- C^* -algebra) is determined by a directed family of submultiplicative $*$ -seminorms (C^* -seminorms). The class of pro- C^* -algebras is bigger than the class of C^* -algebras, for example, $C_{cc}[0, 1]$, the $*$ -algebra of all continuous complex valued functions on $[0, 1]$ with the topology "cc" of uniform convergence on the countable compact subsets of $[0, 1]$, is a pro- C^* -algebra which is not isomorphic to any C^* -algebra. By analogy with the case of Banach $*$ -algebras with bounded approximate unit, A. Inoue (19971) constructed the enveloping pro- C^* -algebra of a pro-Banach $*$ -algebra with bounded approximate unit. S. J. Bhatt and D. J. Karia [Topological algebras with C^* -enveloping algebras, Proc. Indian Acad. Sci (Math. Soc.) 102 (1993), 201-215] proved sufficient and necessary conditions under which a pro-Banach $*$ -algebra with bounded approximate unit admits a C^* -algebra as enveloping pro- C^* -algebra.

A pro-Banach dynamical system is a triple (G, A, α) , where G is a locally compact group, A is a pro-Banach $*$ -algebra with approximate unit and α is a continuous action of G on A (this is, the map $g \rightarrow \alpha_g$ from G to $\text{Aut}(A)$ is a group morphism and the map $g \rightarrow \alpha_g(a)$ from G to A is continuous for each $a \in A$). If the action α is G -invariant (this is, there is a cofinal subset of G -invariant continuous submultiplicative $*$ -seminorms, $p(\alpha_g(a)) \leq M_p p(a)$ for some $M_p > 0$ and for all $a \in A$ and $g \in G$), using the same techniques as in the case of the construction of the crossed product associated to a C^* -dynamical system, we associate a pro- C^* -algebra to a pro-Banach dynamical system, called pro- C^* -crossed product. By definition, the pro- C^* -crossed product associated to a pro-Banach dynamical system (G, A, α) is the enveloping pro- C^* -algebra associated to the covariance algebra $L^1(G, A, \alpha)$.

In this talk, we will discuss about pro-Banach dynamical systems with C^* -crossed products.