Orthogonality preserving property and its stability

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Abstract: In an inner product space $(X, \langle\cdot,\cdot\rangle)$ the natural orthogonality relation $\perp$ can be extended to an ε-orthogonality defined by

$$x\perp^{\varepsilon} y \;\Leftrightarrow\; |\langle x, y\rangle| \leq \varepsilon\|x\|\,\|y\|, \qquad x, y \in X$$

with $\varepsilon \in [0, 1)$.

For inner product spaces X, Y, one can consider the class of (linear) *orthogonality preserving* mappings $f\colon X \to Y$, i.e., those satisfying:

$$x\perp y \;\Rightarrow\; fx\perp fy, \qquad x, y \in X$$

as well as of *approximately orthogonality preserving*:

$$x\perp y \;\Rightarrow\; fx\perp^{\varepsilon} fy, \qquad x, y \in X.$$

The natural problems are: the description of the above classes and the stability, meaning a question whether an approximately orthogonality preserving mapping can be approximated by an orthogonality preserving one.

Generalizations in two directions are possible. First, the scalar-valued inner product can be replaced by a C^*-valued one (C^*-modules). Secondly, one can consider normed spaces with the norm not necessarily coming from the inner product and with some of various possible orthogonality relations.

During the talk we will survey on the recent results in the topic.