BOOK OF ABSTRACTS

Zagreb Dynamical Systems Workshop,

Faculty of Science, Department of Mathematics,
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UNITY THROUGH KNOWLEDGE FUND

MINI-COURSES

Fatou flowers and parabolic curves

Marco Abate, Università di Pisa, Italy

Abstract

A famous result in local complex dynamics in one variable is the Leau-Fatou flower theorem, describing the local dynamics of holomorphic germs tangent to the identity in a whole neighbourhood of the fixed point. The aim of this mini-course is to discuss a few generalisations of this theorem to several complex variables, presenting a few (some very recent) results about the existence of parabolic curves, the multidimensional analogues of the one-dimensional petals of Fatou flowers, and (time permitting) a possible approach for the study of the local dynamics by using the geodesic flow of meromorphic connections on Riemann surfaces.

Thermodynamics for interval maps

Henk Bruin, University of Vienna, Austria

Abstract

The theory of thermodynamic formalism was introduced into dynamics by Sinai, Ruelle and Bowen, initially for hyperbolic systems, to give the ideas of thermodynamic pressure, equilibrium states and phase transitions on a mathematical basis. In the late 1980s mathematicians turned to the non-uniformly hyperbolic case, including intermittent mas (i.e., with neutral fixed points) and unimodal maps. I will discuss the various definitions and shapes of pressure functions for such maps, inducing techniques, and the classification of unimodal maps relevant to this context.

Dynamics of mild dissipative diffeomorphisms of the disc

Sylvain Crovisier, Université Paris-Sud 11, France

Abstract

We discuss a class of volume-contracting surface diffeomorphisms whose dynamics is intermediate between one-dimensional dynamics and general surface dynamics (it includes the dynamics of Hénon diffeomorphisms with Jacobian $< 1/4$). We will see how some classical results for one-dimensional endomorphisms can be pushed into the context of mild dissipative diffeomorphisms. In particular we will present a joint work with Enrique Pujals and Charles Tresser: we describe the dynamics of these systems when the topological entropy vanishes, the structure of the periodic set and its renormalizations.

On the center-focus problem for the equation $\frac{dy}{dx} + \sum_{i=1}^{n} a_i(x) y^i = 0,\ 0 \leq x \leq 1,$ where a_i are polynomials

Lubomir Gavrilov, Université de Toulouse, France

Abstract

We study irreducible components of the center variety, corresponding to the Bautin ideal of the Abel equation

$$\frac{dy}{dx} + \sum_{i=1}^{n} a_i(x) y^i = 0,\ 0 \leq x \leq 1,$$

on the interval $[0, 1]$. A particular attention will be given to components of big co-dimension. As a by-product we obtain the counter-example to the composition conjecture for the center problem (Alwash, 2003), discovered recently by Giné, Grau and Santallusia (2017).

Renato Huzak, Hasselt University, Belgium

Abstract

1. The slow divergence integral on a Mobius band. The slow divergence integral has proved to be an important tool in the study of slow-fast cycles defined on an orientable two-dimensional manifold (e.g. $\mathbb{R}^2$). The goal of our talk is to study 1-canard cycle and 2-canard cycle bifurcations on a non-orientable two-dimensional manifold (e.g. the Mobius band) by using similar techniques. Our focus is on smooth slow-fast models with a Hopf breaking mechanism. The same results can be proved for a jump breaking mechanism and non-generic turning points. The slow-fast bifurcation problems on the Mobius band require the study of the 2-return map attached to such 1- and 2-canard cycles. We give a simple sufficient condition, expressed in terms of the slow divergence integral, for the existence of a period-doubling bifurcation near the 1-canard cycle. We also prove the finite cyclicity property of "singular" 1- and 2-homoclinic loops.

2. Quartic Lienard equations with linear damping. We prove that the quartic Lienard equation with linear damping can have at most two limit cycles, for the parameters kept in a small neighborhood of the origin. Near the origin in the parameter space, the Lienard equation is of singular type and we use singular perturbation theory and the family blow up. To study the limit cycles globally in the phase space we need a suitable Poincare-Lyapunov compactication.

Pavao Mardešić, Université de Bourgogne, France

Abstract

1. **Can one see a dynamical system by looking at some of its orbits?** I present some results by Maja Resman and results from joint work with Maja Resman, Jean Philippe Rolin and Vesna Županović. The general question is: *Can one recognize a dynamical system from the fractal data of its orbits?*

   By fractal data we mean the function $\varepsilon \mapsto A_{z_0}(\epsilon)$ which associates to each $\varepsilon > 0$ the size of the ε-neighborhood of the orbit of the system starting at z_0.

2. **The tennis racket effect and the impossible skateboard flip.** I will present a joint work with physicists D. Sugny and L. Van Damme. We study the phenomenon observed when throwing a tennis racket in the air so that its handle performs a full turn. What one observes is a flip of the tennis racket head. The phenomenon is described by examining Euler's equations. We establish how the phenomenon depends on different parameters.

Complex Cellular Structures (joint with Gal Binyamini)

Dmitry Novikov, Weizmann Institute of Science, Israel

Abstract

Real semialgebraic sets admit so-called cellular decomposition, i.e. representation as a union of cells diffeomorphic to cubes, with some very convenient properties. Attempts to build a straightforward complex holomorphic generalization of this construction meet difficulties related to inner metric properties of holomorphic sets, absent in real case. I will explain these difficulties and explain how a correct generalization can be constructed. This construction is motivated by and related to uniformization results from resolution of singularities theory, by "growth-zeros" ideology of Hilbert 16th problem and by Yomdin-Gromov parameterization results.

https://arxiv.org/abs/1802.07577

The groupoid generated by $PSL(2, C)$ and the exponential and some immiscibility questions

Daniel Panazzolo, Université de Haute Alsace, France

Abstract

I will expose a recent result on the structure of the groupoid of germs of diffeomorphism of the Riemann sphere generated by the Moebius transformations and the exponential. As a byproduct, I show that the subgroup of Homeo$(\mathbb{R}, +\infty)$ (i.e. the germs of real homeomorphism fixing the infinity) which is generated by the positive affine maps $x \mapsto ax + b$ (with $a > 0$), the exponential $x \mapsto \exp(x)$ and the logarithm $x \mapsto \ln(x)$ is isomorphic to a HNN extension (with the exponential playing the role of the stable letter). At the end, I will show how these results are related to several open questions around Hilbert's 16th problem.

Singularities of analytic dynamical systems depending on parameters

Christiane Rousseau, University of Montreal, Canada

Abstract

One basic lesson in bifurcation theory is that the bifurcations of highest codimension in families of dynamical systems depending on parameters organize the bifurcation diagram. Among these, are the bifurcations of equilibrium points, which are studied through normal forms. In analytic families of dynamical systems, the changes of coordinates to normal form generically diverge. The first lecture will illustrate through examples the geometric obstructions to the convergence to normal form. The second lecture will introduce to the geometric methods allowing proving theorems of analytic classification of unfoldings of singularities.

Movable singularities of ODEs: a topological approach

Loïc Teyssier, Université de Strasbourg, France

Abstract

Near a regular point of a complex, scalar and first-order ODE, a local system of solutions defines a trivial fibration disk $\times$ disk. The holomorphic foliation associated to the ODE is obtained by patching up all these local systems, giving a partition of the ambient space into connected Riemann surfaces called leaves (maximal solutions) and singularities of the ODE. There is no reason why this object should continue to be a locally trivial fibration near a singular point, because nothing guarantees that neighboring leaves all have the same topology.

Of course in the simplest example, where the foliation is defined by the level sets of a holomorphic submersion $H : (\mathbb{C}^2, 0) \to (\mathbb{C}, 0)$, the theorem of J. Milnor ensures that H is a holomorphic fibration with total space a well-chosen complement U of the singular fiber $H^{-1}(0)$. In that case the topology of the other leaves is constant, and the natural morphism $\pi_1(H^{-1}(\mathrm{cst})) \to \pi_1(U)$ is injective: the "holes" in the leaves can only be caused by a special set of finitely many "special" leaves (separatrices). Any foliation satisfying this property is deemed *incompressible*.

D. Marín and J.F. Mattei have generalized Milnor theorem to most singular planar holomorphic foliations: under generic assumptions a germ of a foliation is incompressible. After having presented examples of compressible foliations, we will explain the principle of their proof and how

to weaken their assumptions down to an almost sharp characterization of incompressible foliations.

While we understand how to guarantee incompressibility, it is not clear how the extra topology can be accounted for in case of compressibility. We will observe on some examples that it is produced by so-called *movable singularities* (poles of the solutions which do not come from singularities of the ODE). This explanation is satisfying only in a global context, for what does it mean for a solution to “tend to ∞” when the ODE is only defined in a polydisk? In this talk we propose to use compressibility failures to represent persistent movable singularities in a local context, and we will present some (hopefully) convincing arguments in favor of such a definition.

Local classification of differential operators: The “forgotten” case of the local analytic theory of ordinary differential equations

Sergei Yakovenko, Weizmann Institute of Science, Israel

Abstract

Local classification of linear systems of 1st order meromorphic differential equations is a classical subject with most of the results having entered textbooks. Yet in the shadows of this classic theory there hides a very similar but different local theory of higher order linear differential equations. I will explain why this case cannot be considered as a "just a subcase" of the classical theory, present the main results and try to draw attention to surprising similarities between two seemingly different non-commutative algebras. Should be accessible to students from the 3rd year and up.

Renormalization of analytic maps – from rotational domains to parabolic germs

Michael Yampolsky, University of Toronto, Canada

Abstract

I will give a gentle introduction into the powerful renormalization technique, starting with critical maps of the circle, and proceeding to maps with Siegel disks and parabolic germs.

Smooth parametrizations, and their applications in Dynamics and Analysis

Yosef Yomdin, Weizmann Institute of Science, Israel

Abstract

Smooth parametrization consists in a subdivision of a mathematical object under consideration into simple pieces, and then parametric representation of each piece, while keeping control of high order derivatives. Smooth parametrizations have important applications in smooth Dynamics and in Diophantine Geometry.

Main examples for this mini-course are C^k and, (to some extent) analytic parametrizations of semi-algebraic and o-minimal sets.

Very recently in a work of G. Binyamini and D. Novikov a new type of smooth parametrizations was introduced and studied, more powerful that the previously known types (see mini-course of D. Novikov). On this base G. Binyamini and D. Novikov settled a number of outstanding open questions. This includes a conjecture (of 1991) in smooth Dynamics concerning the rate of decay of the tail-entropy of analytic mappings (and many open questions concerning bounding density of rational points on smooth varieties).

We provide a definition of topological entropy $h(f)$ of a mapping f, and of its tail-entropy $h(\epsilon, f)$. We outline the use of smooth parametrizations in bounding $h(f)$, the approach of D. Burguet, G. Liao, J. Yang to bounding $h(\epsilon, f)$, and the result of G. Binyamini and D. Novikov on the complexity of C^k-parametrizations, which finally settles the problem.

Next, we give an overview of some results, open and recently solved problems on smooth parametrizations, from a point of view, which is 'complementary' to the presentation in the mini-course of D. Novikov. Specifically, we consider a special case of smooth parametrization: 'doubling coverings' (or 'conformal invariant Whitney coverings'), and 'Doubling chains'. We present some new results (joint with O. Friedland) on the complexity bounds for doubling coverings, doubling chains, and on the resulting bounds in Kobayashi metric and Doubling inequalities.

50-minute TALKS

Quenched limit theorems for random dynamical systems via spectral method

Davor Dragičević, University of Rijeka, Croatia

Abstract

We will describe how recent advances in the multiplicative ergodic theory can be used to establish limit theorems for a wide class of random dynamical systems. This is a joint work with G. Froyland, C. Gonzalez-Tokman and S. Vaienti.

Complex dimensions and tube formulas

Goran Radunović, University of Zagreb, Croatia

Abstract

We will make a survey of some results of the new theory of complex dimensions for arbitrary subsets of Euclidean spaces of arbitrary dimension. The theory was, for the most part, developed in a series of research papers and in the recent monograph "Fractal Zeta Functions and Fractal Drums: Higher-Dimensional Theory of Complex Dimensions" by the coauthors Michel L. Lapidus, Goran Radunovic and Darko Zubrinic.

The complex dimensions of a given set are defined as poles or more general singularities of the fractal zeta function associated with the set. Two new classes of fractal zeta functions, namely, the distance and the tube zeta function, are introduced and their properties are investigated. The complex dimensions themselves, although defined analytically, have a deep connection with the fractal geometry of the given set. Namely, they play a key role in obtaining the fractal tube formula of a given set, i.e., an asymptotic formula for the Lebesgue measure of the delta neighborhood of the set when delta is close to zero.

Classifications of Dulac germs

Maja Resman, University of Zagreb, Croatia

Abstract

This talk is a continuation of the first course given by P. Mardesic. We describe the structure of the (formal) Fatou coordinate Ψ of a parabolic Dulac germ f satisfying the *Abel equation*:

$$\Psi(f) - \Psi = 1.$$

We demonstrate in which sense is the formal Fatou coordinate the transserial asymptotic expansion of the Fatou coordinate. We ensure the uniqueness of the transserial expansion by introducing the notion of *sectional* expansions related to the choice of an appropriate germ at limit ordinal steps. We also comment on formal and analytic classification of Dulac germs and relation to fractal analysis of their orbits. It is a joint work with P. Mardešić, J. P. Rolin (University of Burgundy) and V. Županović (University of Zagreb).

Variational bounds on entropy of Hamiltonian dynamical systems

Siniša Slijepčević, University of Zagreb, Croatia

Abstract

The problem of understanding of instabilities of Hamiltonian dynamical systems is one of the driving challenges of the theory of dynamical systems. In particular, it is important to describe regions of phase space with positive entropy, and find bounds on entropy and Lyapunov exponents.

In this talk, we will propose a new approach to address it, motivated by recent advances in analysis of evolutionary partial differential equations, which enables new approach to variational formulation of the problem.

As an example of the simplest non-trivial application, we introduce a new measure of instability of area-preserving twist diffeomorphisms, which generalizes the notions of angle of splitting of separatrices, and the flux through a gap of a Cantori. As an example of application, we establish a sharp > 0 lower bound on the topological entropy in a neighbourhood of a hyperbolic, unique action-minimizing fixed point, assuming only no topological obstruction to diffusion, i.e. no homotopically non-trivial invariant circle consisting of orbits with the rotation number 0. The proof is based on a new method of precise construction of positive entropy invariant measures.

We then outline application to more general Lagrangian systems, also in higher degrees of freedom, such as the well-known Arnold's example.

References

[1] Th. Gallay and S. Slijepčević, Distribution of Energy and Convergence to Equilibria in Extended Dissipative Systems, J. Dynam. Differential Equations **27** (2015), 653-682.

[2] S. Slijepčević, A new measure of instability and topological entropy of area-preserving twist diffeomorphisms, Rad HAZU - Mat. Znan. **21** (2017), 117-141.

[3] S. Slijepčević, Variational construction of positive entropy invariant measures of Lagrangian systems and Arnold diffusion, to appear in Ergodic Theory Dynamical Systems (2018)

Lozi-like maps

Sonja Štimac, University of Zagreb, Croatia

Abstract

We define a broad class of piecewise smooth plane homeomorphisms which have properties similar to the properties of Lozi maps, including the existence of a hyperbolic attractor. We call such a map Lozi-like and show that the basic structure of its attractor is determined by the set of its kneading sequences. Joint work with Michal Misiurewicz.

Introduction to fractal analysis of orbits of dynamical systems

Vesna Županović, University of Zagreb, Croatia

Abstract

In this talk I give the initial results concerning analysis of ε- neighborhoods of orbits of dynamical systems. The idea comes from the fractal geometry, while the motivation comes from the 16th Hilbert problem. It is of interest to determine how many limit cycles can bifurcate from a given limit periodic set in a generic unfolding. The cyclicity is classically obtained by studying the multiplicity of fixed points of the Poincaré map. We establish a relation between the cyclicity of a limit periodic set of a planar system and the leading term of the asymptotic expansion of area of ε-neighborhoods of the Poincaré map of an orbit. A natural idea is that higher density of orbits reveals higher cyclicity. The box dimension could be read from the leading term of the asymptotic expansion of area of ε-neighborhood. In this talk I will concentrate on weak focus as a simplest case for the study. Furthermore, shortly I will talk about different directions of research coming from that idea: classifications of Dulac maps, slow-fast systems, oscillatory integrals and fractal zeta functions.