INVITED LECTURES

THURSDAY, 11:00–11:50

Geometric constraints in the level set method for shape and topology optimization

Gregoire Allaire (École Polytechnique)

In the context of structural optimization via a level-set method we propose a novel method to handle geometric constraints related to a notion of local thickness. The local thickness is calculated using the signed distance function to the shape. We formulate global constraints using integral functionals and compute their shape derivatives. We discuss different strategies and possible approximations to handle the geometric constraints. We implement our approach in two and three space dimensions for a model of linearized elasticity. As can be expected, the resulting optimized shapes are strongly dependent on the initial guesses and on the specific treatment of the constraints since, in particular, some topological changes may be prevented by those constraints.

This is a joint work with F. Jouve (Laboratoire J.L.Lions, Université Paris 7) and G. Michailidis (École Polytechnique).

WEDNESDAY

The boundedness-by-entropy principle for cross-diffusion systems from biology

Ansgar Jüngel (Vienna Institute of Tehcnology)

Many systems of collective behavior for multiple species can be described in the continuum limit by cross-diffusion systems, derived e.g. from lattice models. Examples are coming from population dynamics, cell biology, and gas dynamics. A common feature of these strongly coupled parabolic differential equations is that the diffusion matrix is often neither symmetric nor positive definite, which makes the mathematical analysis very challenging.

In this talk, we explain that for certain cross-diffusion systems, these difficulties can be overcome by exploiting a formal gradient-flow structure. This means that there exists a transformation of variables (called entropy variables) such that the transformed diffusion matrix becomes positive definite, and there exists a Lyapunov functional (called entropy) which enables suitable a priori estimates. Although the maximum principle generally does not hold for systems, we show that the entropy concept helps to prove lower and upper bounds for the solutions to certain systems, without the use of a maximum principle. We refer to this technique as the boundedness-by-entropy principle.

We detail the theory for several examples coming from tumor-growth modeling, population dynamics, and multicomponent gas dynamics. The existence of global weak solutions and their long-time behavior is investigated and some numerical examples are presented.

WEDNESDAY, 9:40–10:30

Diffusion-driven unbounded growth and dynamical spike patterns in reaction-diffusion-ode models

Anna Marciniak-Czochra (University of Heidelberg)

In this talk we explore a mechanism of pattern formation arising in processes described by a system of a single reaction-diffusion equation coupled to ordinary differential equations. Such systems of equations arise, for example, in modeling of interactions between cellular processes and diffusing growth factors. Our theory applies to a wide class of pattern formation models with an autocatalytic non-diffusing component.

We show that the lack of diffusion in some model components may lead to singularities which result in instability of all regular stationary patterns. Interestingly, the degeneration of the system yields a continuous spectrum of the linearization operator, which contains positive values. We show that, under some conditions, also all discontinuous stationary solutions are unstable. However, in numerical simulations, solutions having the form of periodic or irregular spikes are observed. We explain this phenomenon using a shadow-type reduction of the reaction-diffusion-ode model. For the resulting system of integro-differential equations, we prove convergence of the model solutions to singular unbounded spike patterns, which location depends on the initial condition.

The talk is a based on a joint research with Kanako Suzuki (Ibaraki University) and Grzegorz Karch (University of Wroclaw).

THURSDAY, 9:40–10:30

Effective interface conditions for the forced infiltration of a viscous fluid into a porous medium using homogenization

Andro Mikelić (Université Lyon 1)

It is generally accepted that the effective velocity of a viscous flow over a porous bed satisfies the Beavers-Joseph slip law. To the contrary, in the case of a forced infiltration of a viscous fluid into a porous medium the interface law has been a subject of controversy.

In this talk, we prove rigorously that the effective interface conditions are:

(i) the continuity of the normal effective velocities;

(ii) zero Darcy's pressure and

(iii) a given slip velocity.

The effective tangential slip velocity is calculated from the boundary layer and depends only on the pore geometry. In the next order of approximation, we derive a pressure slip law. An independent confirmation of the analytical results using direct numerical simulation of the flow at the microscopic level is given, as well.

This is a joint work with Thomas Carraro, Christian Goll and Anna Marciniak-Czochra (all from University of Heidelberg).

THURSDAY, 8:45– 9:35

Towards a quantitative theory in stochastic homogenization

Felix Otto (Max Planck Institute for Mathematics in the Sciences, Leipzig)

In many applications, one has to solve an elliptic partial differential equation with coefficients that vary on a length scale much smaller than the domain size. We are interested in a situation where the coefficients are characterized in *stochastic* terms: Their statistics are assumed to be translation invariant and to decorrelate over large distances. As is known by qualitative theory, the solution operator behaves – on large scales – like the solution operator of an elliptic problem with *homogeneous* deterministic coefficients!

We are interested in several *quantitative* aspects, the first one being: How close is the actual solution to the homogenized one? We give an optimal answer in terms of the quenched Green's function, and point out the connections with elliptic regularity theory (input from Nash's theory, a new outlook on De Giorgi's theory).

We are also interested in the quantitative ergodicity properties for the process called "the environment as seen from the particle". We give an optimal estimate that relies on a link with (the Spectral Gap for) another stochastic process on the coefficient fields, namely heat-bath Glauber dynamics. This connection between statistical mechanics and stochastic homogenization has previously been used in opposite direction (i. e. with qualitative stochastic homogenization as an input).

Theory provides a formula for the homogenized coefficients, based on the construction of a "corrector", which defines harmonic coordinates. This formula has to be approximated in practise, leading to a random and a systematic error. If time permits, we point out optimal estimates of both.

This is joint work with A. Gloria (Université Libre de Bruxelles), S. Neukamm (Weierstraß Institut), and D. Marahrens (Max-Planck-Institute for mathematics in the sciences, Leipzig).

TUESDAY, 11:00–11:50

Method of asymptotic partial decomposition of domain for non-steady problems set in thin rod structures

Grigory Panasenko (Université de Saint-Etienne)

Thin structures are some finite unions of thin rectangles (in 2D settings) or cylinders (in 3D settings) depending on small parameter epsilon $\ll 1$ that is, the ratio of the thickness of the rectangle (cylinder) to its length. The PDE of parabolic or hyperbolic type is considered in thin structures. The asymptotic expansion of the solution is constructed. The error estimates for high order asymptotic approximations are proved. Asymptotic analysis is applied for an asymptotically exact condition of junction of 1D and 2D (or 3D) models.

TUESDAY, 9:40–10:30

Discrete double porosity models

Andrey Piatnitski (Narvik Institute of Technology and Lebedev Physical Institute)

The talk will focus on deriving double-porosity model from simple atomistic interactions on the standard integer lattice. Assuming that the corresponding energy functional consists of finite range strong and weak interaction terms and zero order terms, and that p-growth conditions are fulfilled, we will justify discrete-to-continuous Gamma-convergence results and discuss the properties of the effective model.

TUESDAY, 8:45– 9:35

Anti-Wick and Weyl quantization on ultradistribution spaces

Stevan Pilipović (University of Novi Sad)

We present results related to the global symbol classes so that the corresponding operators act continuously on the space of tempered ultradistributions of Beurling, resp. Roumieu type. For a symbol a belonging to a space of tempered (ultra)distributions, its Anti-Wick quantization is equal to the Weyl quantization of a symbol b given as the convolution of a and the gaussian kernel $e^{-|\cdot|^2}$. The purpose of this talk is twofold. In the first part we give the connection of Anti-Wick and Weyl quantization for symbols belonging to specific symbol classes. The second part is devoted to finding the largest subspace of ultradistributions for which the convolution with the gaussian kernel exist.

This is a joint work with Bojan Prangoski (University of Skopje).

WEDNESDAY, 8:45– 9:35

Interface singularities for the Euler equations

Steve Shkoller (University of Oxford)

I will discuss the problem of interface singularities for the 3-D Euler equations. In the case of one-fluid interfaces, I will discuss the so-called "splash" and "splat" singularities, wherein the crest of a breaking wave crashes onto the trough, and hypersurfaces collide. In the case of two-fluid interfaces, I will explain why such singularities cannot form.

This is joint work with D. Coutand (Heriot-Watt University).

FRIDAY, 8:30– 9:20

Quadratic eigenvalue problem and the reliability of non-selfadjoint spectra

Krešimir Veselić (Fernuniversität Hagen)

We consider the quadratic eigenvalue problem with selfadjoint operator coefficients upon two main applications: (i) relativistic dynamics in the external static field and (ii) damped oscillations of classical mechanical systems. Both models have some mathematical kinship but differ in the interpretation in particular in the role of the spectrum in each of them.

CONTRIBUTED TALKS

THURSDAY, 17:30–18:00

H-distributions via Sobolev spaces

Jelena Aleksić (University of Novi Sad)

H-measures (also known as microlocal defect measures) of Tartar [1] and Gérard [2] obtained for weakly convergent sequences in L^2, and their generalization to L^p, called H-distributions [3], are widely used to determine weather a weakly convergent sequence converge strongly.We extend the concept of H-distributions to the Sobolev spaces and give necessary and sufficient condition so that the weak convergence in $W^{-k,p}$, $p \in (1, \boldsymbol{b})$, implies the strong one.

References

[1] L. Tartar, *H-measures, a new approach for studying homogenisation, oscillations and concentration effects in partial differential equations*, Proc. Roy. Soc. Edinburgh Sect. A **115** (1990), no. 3-4, 193–230.

[2] P. Gérard, *Microlocal defect measures*, Comm. Partial Differential Equations **16** (1991), no. 11, 1761–1794.

[3] N. Antonić and D. Mitrović, *H-distributions: an extension of H-measures to an $L^p - L^q$ setting*, Abstr. Appl. Anal. **2011**, Art. ID 901084, 12 pp.

THURSDAY, 11:55–12:25

On the Hörmander-Mihlin theorem for mixed-norm Lebesgue spaces

Nenad Antonić (University of Zagreb)

For some questions in the theory of partial differential equations, it is useful to consider mixed-norm Lebesgue spaces introduced by Benedek and Panzone (1962). We show that Hörmander's results (1960) on translation invariant operators on Lebesgue spaces extend to the mixed-norm case. We also revisit the Hörmander-Mihlin multiplier theorem in this setting, which was first obtained by Lizorkin (1970), taking particular care to the form of constants in the estimates. This improvement allows us to prove the boundedness of classical pseudodifferential operators on mixed-norm Lebesgue spaces, as well as to generalise the H-distributions, a recently introduced (2011, by Mitrović and the first author) extension of H-measures, to these spaces.

This is a joint work with Ivan Ivec (University of Zagreb).

FRIDAY, 9:25– 9:55

Non-stationary Friedrichs systems

Krešimir Burazin (University of Osijek)

Symmetric positive systems (Friedrichs systems) of first-order linear partial differential equations were introduced by Kurt Otto Friedrichs (1958) in order to treat the equations that change their type, like the equations modelling transonic fluid flow. More recently, Ern, Guermond and Caplain (CPDE, 2007) expressed the theory in terms of operators acting in abstract Hilbert spaces and proved well-posedness result in this abstract setting. Although their setting can be used to represent various boundary value problems for (partial) differential equations, some evolution (non-stationary) problems, such as the initial-boundary value problem for the non-stationary Maxwell system, or the Cauchy problem for the symmetric hyperbolic system, can not be treated within their framework.

We develop an abstract theory for non-stationary Friedrichs systems that can address these problems as well. More precisely, we consider an abstract Cauchy problem in a Hilbert space, that involves a time independent abstract Friedrichs operator. We use the semigroup theory approach, and prove that the operator involved satisfies conditions of the Hille-Yosida generation theorem. We also address the semilinear problem and apply new results to symmetric hyperbolic systems, the unsteady Maxwell system, the unsteady div-grad problem, and the wave equation. The theory can be extended to the complex Banach space setting, with application to the Dirac system.

This is a joint work with Marko Erceg (University of Zagreb).

TUESDAY, 16:35–17:05

The finite difference scheme for 1d flow of a compressible micropolar fluid with homogeneous boundary conditions: a global existence theorem

Nelida Črnjarić-Žic (University of Rijeka)

We define a finite difference method for the nonstationary 1D flow of the compressible viscous and heat-conducting micropolar fluid, assuming that it is in the thermodynamical sense perfect and polytropic. The homogeneous boundary conditions for velocity, microrotation and heat flux are proposed. The sequence of approximate solution for our problem is constructed by using the defined finite difference approximate equations system. We investigate the properties of these approximate solutions and establish their convergence to the strong solution of our problem globally in time. Numerical experiments are performed by solving the defined approximate ordinary differential equations system using strong-stability preserving (SSP) Runge-Kutta scheme for time discretization.

This is a joint work with Nermina Mujaković (University of Rijeka).

TUESDAY, 18:40–19:10

Global existence and uniqueness of the solution for 3-D flow of a compressible viscous micropolar fluid with spherical symmetry

Ivan Dražić (University of Rijeka)

We consider the nonstationary 3-D flow of a compressible viscous heat-conducting micropolar fluid in the domain to be the subset of $\mathbf{R}^3$ bounded with two concentric spheres that present the solid thermo-insulated walls. In the thermodynamical sense, the fluid is perfect and polytropic.

We assume that the initial data are smooth enough and are spherically symmetric functions. The strict positivity for initial density and temperature is also proposed.

Using the fact that the described problem has a generalized solution locally in time we will prove here that the generalized solution exists globally for any time interval $[0, T]$, as well as the uniqueness of the solution.

This is a joint work with Nermina Mujaković (University of Rijeka).

THURSDAY, 18:40–19:10

Localisation principle for 1-scale H-measures

Marko Erceg (University of Zagreb)

Microlocal defect functionals (H-measures, H-distributions, semiclassical measures etc.) are objects which determine, in some sense, the lack of strong compactness for weakly convergent L^p sequences. In contrast to the semiclassical measures, H-measures are not suitable to treat problems with a characteristic length (e.g. thickness of a plate). Luc Tartar overcame the mentioned restriction by introducing 1-scale H-measures, a generalisation of H-measures with a characteristic length.

Moreover, these objects are also an extension of semiclassical measures, being functionals on continuous functions on a compactification of $\mathbf{R}^d \setminus \{0\}$.

We improve and generalise Tartar's localisation principle for 1-scale H-measures from which we are able to derive known localisation principles for H-measures and semiclassical measures. The localisation principle for H-measures has already been successfully applied in many fields (compactness by compensation, small amplitude homogenisation, velocity averaging, averaged control etc.), and the new results are expected to have an even wider class of possible applications.

This is a joint work with Nenad Antonić (University of Zagreb) and Martin Lazar (University of Dubrovnik).

MONDAY, 17:30–18:00

Asymptotic analysis of the polymer fluid flow through a porous medium

Tomislav Fratrović (University of Zagreb)

By its simplicity and its importance in the problems of fluid mechanics, Newton's model plays the major role postulating linear relationship between the viscous stress tensor and the symmetrized gradient of the velocity. In certain cases however, this model is not adequate because the viscosity simply isn't constant, but rather changes significantly with the increased shear stress.

Of all these so-called non-Newtonian or quasi-Newtonian fluids, we turn our attention to the ones obeying the power-law, more precisely the polymer fluids. We observe the flow through a porous domain, a periodic structure in which every cell is made of the fluid part and the impermeable part. The assumption of the periodic nature on the microscale level is fundamental to the asymptotic method called homogenization, whose main task is to determine the global filtration law. This procedure enables for the cumulative effect of the impermeable micro obstacles slowing down the fluid to be described by the effective equations given on the homogeneous domain, obstacle-free.

We show that the results depend on the asymptotic behaviour of one small parameter, the size of the impermeable part, during the homogenization process when number of cells tends to infinity and their size to zero. The obtained specific filtration laws are low volume fraction limit for small size obstacles and nonlinear Brinkman's law in case of the critical obstacle size.

MONDAY, 18:05–18:35

Rod boundary problem: existence and stability of solutions

Mate Kosor (University of Zadar)

Elastic energy of nonlinear hyperelastic curved rod has first been rigorously derived from nonlinear 3D elasticity by Scardia (2006). Based on the orthonormal frames along the reference middle curve (Q) and deformed middle curve (R) the functional can be written as $I_S(Q,R) = \left\| R^T R' - Q^T Q' \right\|^2$, where the norm is equivalent to the usual on $L^2(\langle 0,l\rangle, \mathrm{Skew}(3\times 3))$. Among the assumptions is class C^2 smoothness of the undeformed rod's middle curve. This is a hindrance in some applications.

Rod's behaviour in the point of non-smoothness has been deduced from 3D by Tambača and Velčić (2012). On these grouds, by introducing the unknown $P = RQ^T$, we generalize the elastic energy functional as $I_T(Q,P) = \left\| Q^T P^T P' Q \right\|^2$. This enables the modelling of inextensible rods with Lipschitz middle curve. Complete boundary problem is produced by coupling the elastic energy with the force term and by specifying one or both positions of deformed rod's endpoints. The proof to the existence of the solution uses the usual variational tehniques. When considering a reference rod that can not bridge the two specified endpoints without a deformation, we end up with a fundamental question: is it possible to inextensibly and smoothly deform an arbitrary curve in order to bridge the specified endpoints?

With the new model, we can use a wider range of approximations to the reference geometry: for example, a piecewise straight lines. By the use of Γ-convergence, we show how solutions to the model converge when such approximation to the reference geometry is performed. When constructing the convergent sequence, the geometric questions about curves unavoidably reappear.

This is a joint work with Josip Tambača (University of Zagreb).

THURSDAY, 16:00–16:30

Stability of observations of partial differential equations under uncertain perturbations

Martin Lazar (University of Dubrovnik)

For a differential operator P_1 we analyse the problem of determining the initial energy for the equation $P_1 u = 0$ by observing an additive perturbation of a corresponding solution. It turns out that existing observability results for both the wave and the Schrödinger equation remind stable under robust, additive perturbations determined by a differential operator P_2, which can be chosen almost arbitrary, as well as corresponding initial data.

This generalises the results on averaged control recently obtained by the authors, in which the system under consideration consists of operators of the same type, and the initial data of all the components coincide as well.

The applied techniques employ tools of microlocal analysis, specially the localisation principle for microlocal defect measures or H-measures. However, the mentioned tool turn to be inappropriate when studying a system consisting of two parabolic operators, and a variant, better adapted to a study of parabolic problems is required. This one is found in recently introduced parabolic H-measures, capable of distinguishing different parabolic operators.

Relation of the results to the control theory will be discussed as well.

This is a joint work with Enrique Zuazua (Basque Center for Applied Mathematics, Bilbao).

MONDAY, 16:35–17:05

Penalty immersed boundary method in piezoelectric eel models

Senka Maćešić (University of Rijeka)

In the talk an improved model for energy harvesting piezoelectric eel will be presented. The energy harvesting is achieved by placing an obstacle and a piezoelectric eel in the viscous fluid flow. The obstacle produces Karman vortex street and consequently causes movement of the eel, and bending of the piezoelectric material generates electrical energy. The design of such an energy harvesting system is subject of intense research, and efficient and optimal choices depend on the possibility of accurate mathematical modeling.

The mathematical model consists of two-dimensional incompressible Navier-Stokes equations for the water flow, and two immersed boundaries. First immersed boundary is for the obstacle and can be modeled as a virtual boundary or as a fixed immersed boundary, while second one is for the eel and is modeled as a penalty immersed boundary with non-extendable elastic behavior. Finally, a model for the produced energy also must be included.

The numerical methods are the fractional step method for the incompressible Navier-Stokes equations, with an upwind method for the advection step, and implicit Crank-Nicolson for the diffusion and pressure steps. Furthermore, for the immersed boundaries implicit method is used, and the resulting system of nonlinear equations is solved with some variants of Broyden method. Additionally, an extension of the improvement proposed by LeVeque for the immersed boundary to the penalty immersed boundary method is implemented and new simulation results will be presented.

TUESDAY, 18:05–18:35

General homogenization of a bending-torsion theory for inextensible rods from 3d elasticity

Maroje Marohnić (University of Zagreb)

We derive, by means of Γ-convergence, the equations of homogenized bending rod starting from $3D$ nonlinear elasticity equations. The main assumption is that the energy behaves like h^2 (after dividing by the order h^2 of vanishing volume) where h is the thickness of the body. We do not presuppose any kind of periodicity and work in the general framework. The result shows that, on a subsequence, we always obtain the equations of bending-torsion rod and identifies, in an abstract formulation, the limiting quadratic form connected with that model. This is a generalization from periodic to non-periodic homogenization of bending-torsion rod theory already present in the literature.

This is a joint work with Igor Velčić (University of Zagreb).

TUESDAY, 16:00–16:30

Entropy dissipative approximations in population dynamics

Pina Milišić (University of Zagreb)

In this talk we present novel structure-preserving numerical schemes for Shigesada-Kawasaki-Teramoto (SKT) cross diffusion model from population dynamics. The main features of the proposed discretizations are the preservation of the nonnegativity and the entropy-dissipation structure. For the analysis we combine linear multistep discretizations and G-stability theory, investigated for ODEs from 1980s on, and entropy dissipation methods, which have been proposed in recent years. Entropy dissipation techniques were intensively used in the mathematical analysis of PDEs for derivation of apriori estimates which represent a cruical tool in proving the existence of solutions and studing their long-time behaviour. Our aim was to translate the continuous entropy estimates to the discrete level with hope to obtain more accurate and stable approximations. It is shown that G-stability of the one-leg scheme is sufficient to derive discrete entropy dissipation estimates.

We prove the existence of semi-discrete weak solutions of proposed one-leg multistep time discretizations. Furthermore, under some assumptions on the evolution operator, the second-order convergence of solutions is proved. We note that our results can be applied to some other nonlinear evolution equations as well. In order to underline the theoretical results, few numerical experiments for the population model will be presented.

This is a joint work with Ansgar Jüngel (Technical University of Vienna).

THURSDAY, 18:05–18:35

On H-distributions

Marin Mišur (University of Zagreb)

H-measures were introduced by L. Tartar and independently by P. Gérard to study oscillation and concentration effects in partial differential equations. However, they are suitable only for the problems expressed in the L^2 framework. In order to overcome that limitation, N. Antonić and D. Mitrović introduced H-distributions, as an extension of H-measures to the $\mathrm{L}^p-\mathrm{L}^q$ context. Some existing applications are related to the velocity averaging (M. Lazar and D. Mitrović) and $\mathrm{L}^p-\mathrm{L}^q$ compactness by compensation (M. M. and D. Mitrović).

We shell summarise some recent results from the ongoing work on H-distributions and make a comparison with similar well-known results on H-measures, as well as with recently introduced microlocal compactness forms (F. Rindler). We shall investigate how H-distributions can be used to study concentration effects in partial differential equations.

This is a joint work with Nenad Antonić (University of Zagreb) and Marko Erceg (University of Zagreb).

THURSDAY, 16:35–17:05

On the velocity averaging for equations with optimal heterogeneous rough coefficients

Darko Mitrović (University of Montenegro)

Assume that (u_n) is a sequence of solutions to heterogeneous equations with rough coefficients and fractional derivatives, weakly converging to zero in $L^p(\mathbf{R}^{d+m})$, with $p > 1$. We prove that the sequence of averaged quantities $(\int \rho(\mathbf{y})u_n(\mathbf{x},\mathbf{y})d\mathbf{y})$ is strongly precompact in $L^1_{\mathrm{loc}}(\mathbf{R}^d)$ for any $\rho \in C_c(\mathbf{R}^m)$, provided that restrictive non-degeneracy conditions are satisfied. These are fulfilled for elliptic, parabolic, fractional convection-diffusion equations, as well as for parabolic equations with a fractional time derivative. The main tool that we are using is an adapted version of H-distributions. As a consequence of the introduced methods, we obtain an optimal velocity averaging result in the L^p, $p \geq 2$, framework under the standard non-degeneracy conditions, as well as a connection between the H-measures and the H-distributions.

This is a joint work with Martin Lazar (University of Dubrovnik).

FRIDAY, 11:00–11:30

Regularizing effects of a thin elastic interface in fluid-structure interaction problems

Boris Muha (University of Zagreb)

Motivated by modeling blood flow in human arteries, we study a fluid-structure interaction problem in which the structure is composed of multiple layers, each with possibly different mechanical characteristics and thickness. In the problem presented in this talk the structure is composed of two layers: a thin layer modeled by the 1D wave equation, and a thick layer modeled by the 2D equations of linear elasticity. The flow of an incompressible, viscous fluid is modeled by the Navier-Stokes equations. The thin structure is in contact with the fluid thereby serving as a fluid-structure interface with mass. The coupling between the fluid and the structure is nonlinear. The resulting problem is a nonlinear, moving-boundary problem of parabolic-hyperbolic type.

We show that the model problem has a well-defined energy, and that the energy is bounded by the work done by the inlet and outlet dynamic pressure data. The spaces of weak solutions reveal that the presence of a thin fluid-structure interface with mass regularizes solutions of the coupled problem. We present a constructive proof of an existence of a weak solution for the considered problem.

This is a joint work with S. Čanić (University of Houston).

MONDAY, 16:00–16:30

Error analysis of implicit Runge-Kutta methods and discontinuous Galerkin discretizations for linear Maxwell's equations

Tomislav Pažur (Karlsruhe Institute of Technology)

Maxwell's equations can be considered as an abstract initial value problem

$$u'(t) = Au(t) + f(t), \qquad u(0) = u_0$$

on a suitable Hilbert space H. In this talk we first present an error analysis for Gauß and Radau collocation methods applied to this abstract problem. Our error analysis is based on energy technique discussed in [1]. For s-stage collocation methods we obtain an order reduction to order $s+1$ instead of the classical order $2s$ and $2s-1$ for Gauß and Radau collocation methods, respectively.

Next we discretize Maxwell's equations in space using the discontinuous Galerkin method and then apply a collocation method to integrate the semidiscrete problem in time. We can prove that the full discretization error is of size $\mathcal{O}(h^{p+1/2} + \tau^{s+1})$, where h denotes maximum diameter of the finite elements and τ denotes the time step. Finally, we illustrate our theoretical results by numerical experiments.

This is a joint work with Marlis Hochbruck (Karlsruhe Institute of Technology).

References

[1] C. Lubich and A. Ostermann, *Runge-Kutta approximation of quasi-linear parabolic equations*, Mathematics of Computation **64** (1995), No. 210, 601–628.

TUESDAY, 11:55–12:25

A shell model for shells with little regularity

Josip Tambača (University of Zagreb)

In this talk, a linear shell model of Koiter's type will be formulated. The model captures the membrane and bending effects and is well formulated for shells whose middle surface is parameterized by a $W^{1,\infty}$ function.

Unknowns in the model are the displacement $\tilde{\boldsymbol{u}}$ of the middle surface of the shell and the infinitesimal rotation $\tilde{\boldsymbol{\omega}}$ of the shell cross-section. The existence and uniqueness of the solution $(\tilde{\boldsymbol{u}}, \tilde{\boldsymbol{\omega}}) \in H^1 \times H^1$ easily follows by the Lax-Milgram lemma.

The model is analyzed asymptotically with respect to the small thickness of the shell. It is shown that the model asymptotically behaves just as the membrane model, the flexural model, and the generalized membrane model in each regime.

Since the model is well formulated for shells whose middle surface is with corners, the model is compared with Le Dret's model of folded plates. The model is also compared with the Cosserat shell model with a single director.

FRIDAY, 11:35–12:05

Simultaneous homogenization and dimensional reduction in elasticity

Igor Velčić (University of Zagreb)

Starting from $3D$ elasticity equations we discuss the derivation of plate models by means of Gamma convergence taking into account the effects of simultaneous homogenization and dimensional reduction. We discuss bending plate and von Kárman plate model obtained in this way. The obtained models depend on the ratio between two small parameters; h being the thickness of the body and $\varepsilon(h)$ being the oscillations of the material.

Part of this work is collaboration with P. Hornung (University of Bonn) and S. Neukamm (Weierstraß Institut).

TUESDAY, 17:30–18:00

Fully homogenized model for immiscible incompressible two-phase flow through heterogeneous porous media with thin fractures

Anja Vrbaški (University of Zagreb)

We derive a model describing global behavior of the two-phase incompressible flow in fractured porous media. The fractured media is regarded as a porous medium consisting of two superimposed continua, a connected fracture system, which is assumed to be thin of order $\varepsilon\delta$, and an ε–periodic system of disjoint matrix blocks. Global behavior of the fractured media is derived by passing to the limit as $\varepsilon \to 0$ and then as the relative fracture thickness $\delta \to 0$, taking into account that the permeability of the blocks is proportional to $(\varepsilon\delta)^2$, while permeability of the fractures is of order one. The resulting macroscopic model is a fully homogenized model, that is, where all the coefficients are calculated in terms of given data and do not depend on the additional coupling or cell problems.

This is a joint work with Mladen Jurak (University of Zagreb) and Leonid Pankratov (Institute for Low Temperature Physics and Engineering, Kharkov).

FRIDAY, 10:00–10:30

Classical solutions in optimal design problems

Marko Vrdoljak (University of Zagreb)

In optimal design problems, one is trying to find the best arrangement of given materials, such that the obtained body has some optimal properties. The optimality of an arrangement is measured by a cost functional, which is usually an integral functional depending on the distribution of materials and the state function, obtained as a solution of the associated boundary value problem (the state equation).

Commonly, optimal design problems do not have solutions (we refer to such solutions as *classical*), so one considers proper relaxations of original problems. Relaxation by the homogenization method consists in introducing generalized materials, which are mixtures of original materials on the micro-scale.

We consider optimal design problems for stationary diffusion in the case of two isotropic phases, with several state equations, and a convex combination of compliances as the cost functional. If domain is spherically symmetric, and the right-hand sides of state equations are radial functions, we are able to show that there exists an optimal (relaxed) design which is a radial function. As a consequence, we can write down a simpler relaxation of the original optimal design problem, whose necessary conditions of optimality enable us to calculate a radial optimal design. By using this method, we shall present some problems having classical solutions.

MONDAY, 18:40–19:10

One dimensional model of biodegradable elastic curved rods

Bojan Žugec (University of Zagreb)

In this work we consider a curved elastic rod made of biodegradable material situated in a liquid solvent. By asymptotic expansion we derive a one dimensional model of biodegradable elastic curved rod from the associated 3D problem. The model is described by two coupled equations describing behaviour of the linearized elastic curved rod and the evolution of the rods material at time t. Two cases of biodegradation are being observed: the case where diffusion has an effect on the process of biodegradation, i.e. material loss and the case where it has not. In both of these cases the existence and uniqueness of the one dimensional model solution will be proved.

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