Matematički odjel
Prirodoslovno–matematički fakultet
Sveučilište u Zagrebu

Vedran Krčadinac

Konstrukcija konačnih ravnina u programskom sustavu *Mathematica*

ILUSTRACIJE I PRIMJERI

Zagreb, svibanj 1996.

Format u kojem se pohranjuju konačne ravnine u *Mathematici*

Konačna projektivna ravnina reda tri:

```
{{2, 5, 8, 11}, {1, 5, 6, 7}, {4, 5, 10, 12}, {3, 5, 9, 13}, {1, 2, 3, 4},
 {2, 7, 10, 13}, {2, 6, 9, 12}, {1, 11, 12, 13}, {4, 7, 9, 11}, {3, 6, 10, 11},
 {1, 8, 9, 10},  {3, 7, 8, 12}, {4, 6, 8, 13}}
```

Konačna afina ravnina reda četiri:

-nenormalizirana (točke su uređeni parovi)

```
{{{{0, 0}, {0, 1}, {0, 2}, {0, 3}}, {{1, 0}, {1, 1}, {1, 2}, {1, 3}},
   {{2, 0}, {2, 1}, {2, 2}, {2, 3}}, {{3, 0}, {3, 1}, {3, 2}, {3, 3}}},
  {{{0, 0}, {1, 0}, {2, 0}, {3, 0}}, {{0, 1}, {1, 1}, {2, 1}, {3, 1}},
   {{0, 2}, {1, 2}, {2, 2}, {3, 2}}, {{0, 3}, {1, 3}, {2, 3}, {3, 3}}},
  {{{0, 0}, {1, 1}, {2, 2}, {3, 3}}, {{0, 1}, {1, 0}, {2, 3}, {3, 2}},
   {{0, 2}, {1, 3}, {2, 0}, {3, 1}}, {{0, 3}, {1, 2}, {2, 1}, {3, 0}}},
  {{{0, 0}, {1, 2}, {2, 3}, {3, 1}}, {{0, 1}, {1, 3}, {2, 2}, {3, 0}},
   {{0, 2}, {1, 0}, {2, 1}, {3, 3}}, {{0, 3}, {1, 1}, {2, 0}, {3, 2}}},
  {{{0, 0}, {1, 3}, {2, 1}, {3, 2}}, {{0, 1}, {1, 2}, {2, 0}, {3, 3}},
   {{0, 2}, {1, 1}, {2, 3}, {3, 0}}, {{0, 3}, {1, 0}, {2, 2}, {3, 1}}}}
```

-normalizirana (točke su brojevi $1 \ldots 16$)

```
{{{1, 5, 9, 13}, {2, 6, 10, 14}, {3, 7, 11, 15}, {4, 8, 12, 16}},
  {{1, 2, 3, 4}, {5, 6, 7, 8}, {9, 10, 11, 12}, {13, 14, 15, 16}},
  {{1, 6, 11, 16}, {5, 2, 15, 12}, {9, 14, 3, 8}, {13, 10, 7, 4}},
  {{1, 10, 15, 8}, {5, 14, 11, 4}, {9, 2, 7, 16}, {13, 6, 3, 12}},
  {{1, 14, 7, 12}, {5, 10, 3, 16}, {9, 6, 15, 4}, {13, 2, 11, 8}}}
```

Pravci su grupirani u klase međusobno paralelnih.

Desarguesova projektivna ravnina reda devet:

```
{{2, 11, 20, 29, 38, 47, 56, 65, 74, 83}, {1, 11, 12, 13, 14, 15, 16, 17, 18, 19},
 {4, 11, 22, 30, 44, 55, 63, 68, 79, 87}, {3, 11, 21, 31, 41, 51, 61, 71, 81, 91},
 {10, 11, 28, 33, 40, 54, 59, 66, 80, 88}, {8, 11, 26, 32, 43, 49, 64, 72, 78, 84},
 {7, 11, 25, 36, 42, 53, 58, 73, 75, 86}, {6, 11, 24, 37, 39, 52, 62, 67, 77, 90},
 {9, 11, 27, 34, 46, 50, 57, 69, 76, 89}, {5, 11, 23, 35, 45, 48, 60, 70, 82, 85},
 {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}, {2, 13, 22, 31, 40, 49, 58, 67, 76, 85},
 {2, 12, 21, 30, 39, 48, 57, 66, 75, 84}, {2, 19, 28, 37, 46, 55, 64, 73, 82, 91},
 {2, 17, 26, 35, 44, 53, 62, 71, 80, 89}, {2, 16, 25, 34, 43, 52, 61, 70, 79, 88},
 {2, 15, 24, 33, 42, 51, 60, 69, 78, 87}, {2, 18, 27, 36, 45, 54, 63, 72, 81, 90},
 {2, 14, 23, 32, 41, 50, 59, 68, 77, 86}, {1, 29, 30, 31, 32, 33, 34, 35, 36, 37},
 {4, 13, 21, 29, 46, 54, 62, 70, 78, 86}, {3, 12, 22, 29, 42, 52, 59, 72, 82, 89},
 {10, 19, 24, 29, 45, 50, 58, 71, 79, 84}, {8, 17, 23, 29, 40, 55, 61, 69, 75, 90},
 {7, 16, 27, 29, 44, 49, 60, 66, 77, 91}, {6, 15, 28, 29, 43, 53, 57, 68, 81, 85},
 {9, 18, 25, 29, 41, 48, 64, 67, 80, 87}, {5, 14, 26, 29, 39, 51, 63, 73, 76, 88},
 {1, 20, 21, 22, 23, 24, 25, 26, 27, 28}, {3, 13, 20, 30, 43, 50, 60, 73, 80, 90},
 {4, 12, 20, 31, 45, 53, 64, 69, 77, 88}, {6, 19, 20, 33, 44, 48, 61, 72, 76, 86},
 {5, 17, 20, 32, 42, 54, 57, 67, 79, 91}, {9, 16, 20, 36, 39, 55, 59, 71, 78, 85},
 {10, 15, 20, 37, 41, 49, 63, 70, 75, 89}, {7, 18, 20, 34, 40, 51, 62, 68, 82, 84},
 {8, 14, 20, 35, 46, 52, 58, 66, 81, 87}, {1, 83, 84, 85, 86, 87, 88, 89, 90, 91},
 {8, 13, 28, 34, 42, 48, 63, 71, 77, 83}, {5, 12, 24, 36, 46, 49, 61, 68, 80, 83},
 {4, 19, 27, 35, 43, 51, 59, 67, 75, 83}, {7, 17, 22, 33, 39, 50, 64, 70, 81, 83},
 {6, 16, 26, 30, 41, 54, 58, 69, 82, 83}, {3, 15, 25, 32, 45, 55, 62, 66, 76, 83},
 {10, 18, 23, 31, 44, 52, 57, 73, 78, 83}, {9, 14, 21, 37, 40, 53, 60, 72, 79, 83},
 {1, 65, 66, 67, 68, 69, 70, 71, 72, 73}, {10, 13, 27, 32, 39, 53, 61, 65, 82, 87},
 {6, 12, 25, 35, 40, 50, 63, 65, 78, 91}, {9, 19, 23, 30, 42, 49, 62, 65, 81, 88},
 {4, 17, 28, 36, 41, 52, 60, 65, 76, 84}, {8, 16, 22, 37, 45, 51, 57, 65, 80, 86},
 {7, 15, 26, 31, 46, 48, 59, 65, 79, 90}, {5, 18, 21, 33, 43, 55, 58, 65, 77, 89},
 {3, 14, 24, 34, 44, 54, 64, 65, 75, 85}, {1, 56, 57, 58, 59, 60, 61, 62, 63, 64},
 {9, 13, 26, 33, 45, 52, 56, 68, 75, 91}, {7, 12, 23, 37, 43, 54, 56, 71, 76, 87},
 {5, 19, 22, 34, 41, 53, 56, 66, 78, 90}, {10, 17, 25, 30, 46, 51, 56, 72, 77, 85},
 {4, 16, 24, 32, 40, 48, 56, 73, 81, 89}, {8, 15, 21, 36, 44, 50, 56, 67, 82, 88},
 {3, 18, 28, 35, 39, 49, 56, 69, 79, 86}, {6, 14, 27, 31, 42, 55, 56, 70, 80, 84},
 {1, 47, 48, 49, 50, 51, 52, 53, 54, 55}, {5, 13, 25, 37, 44, 47, 59, 69, 81, 84},
 {8, 12, 27, 33, 41, 47, 62, 73, 79, 85}, {3, 19, 26, 36, 40, 47, 57, 70, 77, 87},
 {9, 17, 24, 31, 43, 47, 63, 66, 82, 86}, {10, 16, 21, 35, 42, 47, 64, 68, 76, 90},
 {4, 15, 23, 34, 39, 47, 58, 72, 80, 91}, {6, 18, 22, 32, 46, 47, 60, 71, 75, 88},
 {7, 14, 28, 30, 45, 47, 61, 67, 78, 89}, {1, 74, 75, 76, 77, 78, 79, 80, 81, 82},
 {7, 13, 24, 35, 41, 55, 57, 72, 74, 88}, {9, 12, 28, 32, 44, 51, 58, 70, 74, 90},
 {8, 19, 25, 31, 39, 54, 60, 68, 74, 89}, {6, 17, 21, 34, 45, 49, 59, 73, 74, 87},
 {3, 16, 23, 33, 46, 53, 63, 67, 74, 84}, {5, 15, 27, 30, 40, 52, 64, 71, 74, 86},
 {4, 18, 26, 37, 42, 50, 61, 66, 74, 85}, {10, 14, 22, 36, 43, 48, 62, 69, 74, 91},
 {1, 38, 39, 40, 41, 42, 43, 44, 45, 46}, {6, 13, 23, 36, 38, 51, 64, 66, 79, 89},
 {10, 12, 26, 34, 38, 55, 60, 67, 81, 86}, {7, 19, 21, 32, 38, 52, 63, 69, 80, 85},
 {3, 17, 27, 37, 38, 48, 58, 68, 78, 88}, {5, 16, 28, 31, 38, 50, 62, 72, 75, 87},
 {9, 15, 22, 35, 38, 54, 61, 73, 77, 84}, {8, 18, 24, 30, 38, 53, 59, 70, 76, 91},
 {4, 14, 25, 33, 38, 49, 57, 71, 82, 90}}
```

Translacijska projektivna ravnina reda devet:

```
{{2, 11, 20, 29, 38, 47, 56, 65, 74, 83}, {1, 11, 12, 13, 14, 15, 16, 17, 18, 19},
 {4, 11, 22, 30, 44, 55, 63, 68, 79, 87}, {3, 11, 21, 31, 41, 51, 61, 71, 81, 91},
 {5, 11, 23, 35, 40, 52, 64, 66, 78, 90}, {6, 11, 24, 37, 45, 49, 59, 70, 80, 84},
 {7, 11, 25, 36, 42, 53, 58, 73, 75, 86}, {8, 11, 26, 32, 39, 54, 60, 67, 82, 88},
 {9, 11, 27, 34, 46, 50, 57, 69, 76, 89}, {10, 11, 28, 33, 43, 48, 62, 72, 77, 85},
 {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}, {2, 13, 22, 31, 40, 49, 58, 67, 76, 85},
 {2, 12, 21, 30, 39, 48, 57, 66, 75, 84}, {2, 14, 23, 32, 41, 50, 59, 68, 77, 86},
 {2, 15, 24, 33, 42, 51, 60, 69, 78, 87}, {2, 16, 25, 34, 43, 52, 61, 70, 79, 88},
 {2, 17, 26, 35, 44, 53, 62, 71, 80, 89}, {2, 18, 27, 36, 45, 54, 63, 72, 81, 90},
 {2, 19, 28, 37, 46, 55, 64, 73, 82, 91}, {1, 29, 30, 31, 32, 33, 34, 35, 36, 37},
 {4, 13, 21, 29, 46, 54, 62, 70, 78, 86}, {3, 12, 22, 29, 42, 52, 59, 72, 82, 89},
 {5, 14, 26, 29, 43, 55, 58, 69, 81, 84}, {6, 15, 28, 29, 40, 50, 63, 71, 75, 88},
 {7, 16, 27, 29, 44, 49, 60, 66, 77, 91}, {8, 17, 23, 29, 45, 51, 57, 73, 79, 85},
 {9, 18, 25, 29, 41, 48, 64, 67, 80, 87}, {10, 19, 24, 29, 39, 53, 61, 68, 76, 90},
 {1, 20, 21, 22, 23, 24, 25, 26, 27, 28}, {3, 13, 20, 30, 43, 50, 60, 73, 80, 90},
 {4, 12, 20, 31, 45, 53, 64, 69, 77, 88}, {8, 14, 20, 35, 42, 48, 63, 70, 76, 91},
 {10, 15, 20, 37, 44, 52, 57, 67, 81, 86}, {9, 16, 20, 36, 39, 55, 59, 71, 78, 85},
 {5, 17, 20, 32, 46, 49, 61, 72, 75, 87}, {7, 18, 20, 34, 40, 51, 62, 68, 82, 84},
 {6, 19, 20, 33, 41, 54, 58, 66, 79, 89}, {1, 38, 39, 40, 41, 42, 43, 44, 45, 46},
 {8, 13, 28, 34, 38, 53, 59, 66, 81, 87}, {5, 12, 24, 36, 38, 50, 62, 67, 79, 91},
 {4, 14, 25, 33, 38, 49, 57, 71, 82, 90}, {9, 15, 22, 35, 38, 54, 61, 73, 77, 84},
 {6, 16, 26, 30, 38, 51, 64, 72, 76, 86}, {3, 17, 27, 37, 38, 48, 58, 68, 78, 88},
 {10, 18, 23, 31, 38, 55, 60, 70, 75, 89}, {7, 19, 21, 32, 38, 52, 63, 69, 80, 85},
 {1, 47, 48, 49, 50, 51, 52, 53, 54, 55}, {10, 13, 27, 32, 42, 47, 64, 71, 79, 84},
 {6, 12, 25, 35, 46, 47, 60, 68, 81, 85}, {7, 14, 28, 30, 45, 47, 61, 67, 78, 89},
 {4, 15, 23, 34, 39, 47, 58, 72, 80, 91}, {8, 16, 22, 37, 41, 47, 62, 69, 75, 90},
 {9, 17, 24, 31, 43, 47, 63, 66, 82, 86}, {5, 18, 21, 33, 44, 47, 59, 73, 76, 88},
 {3, 19, 26, 36, 40, 47, 57, 70, 77, 87}, {1, 56, 57, 58, 59, 60, 61, 62, 63, 64},
 {9, 13, 26, 33, 45, 52, 56, 68, 75, 91}, {7, 12, 23, 37, 43, 54, 56, 71, 76, 87},
 {10, 14, 22, 36, 46, 51, 56, 66, 80, 88}, {5, 15, 27, 30, 41, 53, 56, 70, 82, 85},
 {4, 16, 24, 32, 40, 48, 56, 73, 81, 89}, {6, 17, 21, 34, 42, 55, 56, 67, 77, 90},
 {3, 18, 28, 35, 39, 49, 56, 69, 79, 86}, {8, 19, 25, 31, 44, 50, 56, 72, 78, 84},
 {1, 65, 66, 67, 68, 69, 70, 71, 72, 73}, {5, 13, 25, 37, 39, 51, 63, 65, 77, 89},
 {8, 12, 27, 33, 40, 55, 61, 65, 80, 86}, {3, 14, 24, 34, 44, 54, 64, 65, 75, 85},
 {7, 15, 26, 31, 46, 48, 59, 65, 79, 90}, {10, 16, 21, 35, 45, 50, 58, 65, 82, 87},
 {4, 17, 28, 36, 41, 52, 60, 65, 76, 84}, {6, 18, 22, 32, 43, 53, 57, 65, 78, 91},
 {9, 19, 23, 30, 42, 49, 62, 65, 81, 88}, {1, 74, 75, 76, 77, 78, 79, 80, 81, 82},
 {7, 13, 24, 35, 41, 55, 57, 72, 74, 88}, {9, 12, 28, 32, 44, 51, 58, 70, 74, 90},
 {6, 14, 27, 31, 39, 52, 62, 73, 74, 87}, {8, 15, 21, 36, 43, 49, 64, 68, 74, 89},
 {3, 16, 23, 33, 46, 53, 63, 67, 74, 84}, {10, 17, 25, 30, 40, 54, 59, 69, 74, 91},
 {4, 18, 26, 37, 42, 50, 61, 66, 74, 85}, {5, 19, 22, 34, 45, 48, 60, 71, 74, 86},
 {1, 83, 84, 85, 86, 87, 88, 89, 90, 91}, {6, 13, 23, 36, 44, 48, 61, 69, 82, 83},
 {10, 12, 26, 34, 41, 49, 63, 73, 78, 83}, {9, 14, 21, 37, 40, 53, 60, 72, 79, 83},
 {3, 15, 25, 32, 45, 55, 62, 66, 76, 83}, {5, 16, 28, 31, 42, 54, 57, 68, 80, 83},
 {7, 17, 22, 33, 39, 50, 64, 70, 81, 83}, {8, 18, 24, 30, 46, 52, 58, 71, 77, 83},
 {4, 19, 27, 35, 43, 51, 59, 67, 75, 83}}
```

Translacijski pravac je jedanaesti po redu, $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$.

Dualna translacijska projektivna ravnina reda devet:

```
{{2, 11, 20, 29, 38, 47, 56, 65, 74, 83}, {1, 11, 12, 13, 14, 15, 16, 17, 18, 19},
 {4, 11, 22, 30, 44, 55, 63, 68, 79, 87}, {3, 11, 21, 31, 41, 51, 61, 71, 81, 91},
 {5, 11, 23, 35, 40, 54, 60, 66, 82, 88}, {6, 11, 24, 37, 43, 49, 62, 72, 77, 84},
 {7, 11, 25, 36, 46, 50, 58, 69, 75, 89}, {8, 11, 26, 32, 39, 52, 64, 67, 78, 90},
 {9, 11, 27, 34, 42, 53, 57, 73, 76, 86}, {10, 11, 28, 33, 45, 48, 59, 70, 80, 85},
 {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}, {2, 13, 22, 31, 40, 49, 58, 67, 76, 85},
 {2, 12, 21, 30, 39, 48, 57, 66, 75, 84}, {2, 14, 23, 32, 41, 50, 59, 68, 77, 86},
 {2, 15, 24, 33, 42, 51, 60, 69, 78, 87}, {2, 16, 25, 34, 43, 52, 61, 70, 79, 88},
 {2, 17, 26, 35, 44, 53, 62, 71, 80, 89}, {2, 18, 27, 36, 45, 54, 63, 72, 81, 90},
 {2, 19, 28, 37, 46, 55, 64, 73, 82, 91}, {1, 29, 30, 31, 32, 33, 34, 35, 36, 37},
 {4, 13, 21, 29, 46, 54, 62, 70, 78, 86}, {3, 12, 22, 29, 42, 52, 59, 72, 82, 89},
 {5, 14, 26, 29, 45, 51, 58, 73, 79, 84}, {6, 15, 28, 29, 40, 53, 61, 68, 75, 90},
 {7, 16, 27, 29, 41, 49, 64, 66, 80, 87}, {8, 17, 23, 29, 43, 55, 57, 69, 81, 85},
 {9, 18, 25, 29, 44, 48, 60, 67, 77, 91}, {10, 19, 24, 29, 39, 50, 63, 71, 76, 88},
 {1, 20, 21, 22, 23, 24, 25, 26, 27, 28}, {3, 13, 20, 30, 43, 50, 60, 73, 80, 90},
 {4, 12, 20, 31, 45, 53, 64, 69, 77, 88}, {8, 14, 20, 35, 46, 48, 61, 72, 76, 87},
 {10, 15, 20, 37, 41, 54, 57, 67, 79, 89}, {9, 16, 20, 36, 39, 51, 62, 68, 82, 85},
 {5, 17, 20, 32, 42, 49, 63, 70, 75, 91}, {7, 18, 20, 34, 40, 55, 59, 71, 78, 84},
 {6, 19, 20, 33, 44, 52, 58, 66, 81, 86}, {1, 38, 39, 40, 41, 42, 43, 44, 45, 46},
 {8, 13, 28, 34, 38, 51, 63, 66, 77, 89}, {5, 12, 24, 36, 38, 55, 61, 67, 80, 86},
 {4, 14, 27, 37, 38, 52, 60, 71, 75, 85}, {7, 15, 22, 32, 38, 48, 62, 73, 81, 88},
 {10, 16, 23, 30, 38, 53, 58, 72, 78, 91}, {3, 17, 25, 33, 38, 54, 64, 68, 76, 84},
 {6, 18, 26, 31, 38, 50, 57, 70, 82, 87}, {9, 19, 21, 35, 38, 49, 59, 69, 79, 90},
 {1, 47, 48, 49, 50, 51, 52, 53, 54, 55}, {10, 13, 27, 32, 44, 47, 61, 69, 82, 84},
 {6, 12, 25, 35, 41, 47, 63, 73, 78, 85}, {9, 14, 24, 30, 40, 47, 64, 70, 81, 89},
 {4, 15, 26, 36, 43, 47, 59, 66, 76, 91}, {5, 16, 22, 33, 46, 47, 57, 71, 77, 90},
 {7, 17, 28, 31, 39, 47, 60, 72, 79, 86}, {8, 18, 21, 37, 42, 47, 58, 68, 80, 88},
 {3, 19, 23, 34, 45, 47, 62, 67, 75, 87}, {1, 56, 57, 58, 59, 60, 61, 62, 63, 64},
 {9, 13, 26, 33, 41, 55, 56, 72, 75, 88}, {7, 12, 23, 37, 44, 51, 56, 70, 76, 90},
 {6, 14, 22, 34, 39, 54, 56, 69, 80, 91}, {8, 15, 25, 30, 45, 49, 56, 71, 82, 86},
 {4, 16, 28, 35, 42, 50, 56, 67, 81, 84}, {10, 17, 21, 36, 40, 52, 56, 73, 77, 87},
 {3, 18, 24, 32, 46, 53, 56, 66, 79, 85}, {5, 19, 27, 31, 43, 48, 56, 68, 78, 89},
 {1, 65, 66, 67, 68, 69, 70, 71, 72, 73}, {5, 13, 25, 37, 39, 53, 59, 65, 81, 87},
 {8, 12, 27, 33, 40, 50, 62, 65, 79, 91}, {3, 14, 28, 36, 44, 49, 57, 65, 78, 88},
 {9, 15, 23, 31, 46, 52, 63, 65, 80, 84}, {6, 16, 21, 32, 45, 55, 60, 65, 76, 89},
 {4, 17, 24, 34, 41, 48, 58, 65, 82, 90}, {10, 18, 22, 35, 43, 51, 64, 65, 75, 86},
 {7, 19, 26, 30, 42, 54, 61, 65, 77, 85}, {1, 74, 75, 76, 77, 78, 79, 80, 81, 82},
 {7, 13, 24, 35, 45, 52, 57, 68, 74, 91}, {9, 12, 28, 32, 43, 54, 58, 71, 74, 87},
 {10, 14, 25, 31, 42, 55, 62, 66, 74, 90}, {5, 15, 21, 34, 44, 50, 64, 72, 74, 85},
 {3, 16, 26, 37, 40, 48, 63, 69, 74, 86}, {6, 17, 27, 30, 46, 51, 59, 67, 74, 88},
 {4, 18, 23, 33, 39, 49, 61, 73, 74, 89}, {8, 19, 22, 36, 41, 53, 60, 70, 74, 84},
 {1, 83, 84, 85, 86, 87, 88, 89, 90, 91}, {6, 13, 23, 36, 42, 48, 64, 71, 79, 83},
 {10, 12, 26, 34, 46, 49, 60, 68, 81, 83}, {7, 14, 21, 33, 43, 53, 63, 67, 82, 83},
 {3, 15, 27, 35, 39, 55, 58, 70, 77, 83}, {8, 16, 24, 31, 44, 54, 59, 73, 75, 83},
 {9, 17, 22, 37, 45, 50, 61, 66, 78, 83}, {5, 18, 28, 30, 41, 52, 62, 69, 76, 83},
 {4, 19, 25, 32, 40, 51, 57, 72, 80, 83}}
```

Translacijska točka je `11`.

Hughesova projektivna ravnina reda devet:

```
{{4, 13, 21, 29, 46, 54, 62, 70, 78, 86}, {29, 1, 31, 30, 32, 33, 34, 35, 36, 37},
 {30, 2, 12, 21, 84, 75, 66, 57, 48, 39}, {21, 11, 3, 31, 91, 81, 71, 61, 51, 41},
 {31, 4, 20, 12, 64, 45, 53, 88, 69, 77}, {12, 29, 22, 3, 52, 59, 42, 82, 89, 72},
 {3, 30, 13, 20, 60, 43, 50, 90, 73, 80}, {20, 21, 1, 22, 25, 23, 24, 28, 26, 27},
 {22, 31, 2, 13, 40, 49, 58, 67, 76, 85}, {13, 12, 11, 1, 14, 15, 16, 17, 18, 19},
 {1, 3, 4, 2, 6, 7, 5, 9, 10, 8}, {2, 20, 29, 11, 47, 56, 38, 74, 83, 65},
 {11, 22, 30, 4, 87, 79, 68, 63, 55, 44}, {8, 13, 28, 34, 38, 51, 63, 66, 77, 89},
 {10, 13, 27, 32, 44, 47, 61, 69, 82, 84}, {9, 13, 26, 33, 41, 55, 56, 72, 75, 88},
 {5, 13, 25, 37, 39, 53, 59, 65, 81, 87}, {7, 13, 24, 35, 45, 52, 57, 68, 74, 91},
 {6, 13, 23, 36, 42, 48, 64, 71, 79, 83}, {65, 1, 67, 66, 68, 69, 70, 71, 72, 73},
 {83, 1, 85, 84, 86, 87, 88, 89, 90, 91}, {74, 1, 76, 75, 77, 78, 79, 80, 81, 82},
 {38, 1, 40, 39, 41, 42, 43, 44, 45, 46}, {56, 1, 58, 57, 59, 60, 61, 62, 63, 64},
 {47, 1, 49, 48, 50, 51, 52, 53, 54, 55}, {44, 2, 17, 71, 62, 89, 35, 53, 80, 26},
 {55, 2, 19, 91, 37, 46, 82, 73, 28, 64}, {63, 2, 18, 81, 72, 36, 54, 27, 45, 90},
 {68, 2, 14, 41, 77, 50, 23, 86, 59, 32}, {79, 2, 16, 61, 43, 25, 88, 34, 70, 52},
 {87, 2, 15, 51, 24, 69, 60, 42, 33, 78}, {86, 11, 9, 53, 34, 73, 57, 42, 27, 76},
 {78, 11, 8, 64, 39, 32, 90, 26, 67, 52}, {70, 11, 10, 45, 80, 48, 33, 85, 59, 28},
 {62, 11, 6, 77, 72, 24, 49, 37, 43, 84}, {54, 11, 5, 88, 23, 40, 82, 66, 35, 60},
 {46, 11, 7, 69, 58, 89, 25, 50, 75, 36}, {37, 4, 74, 42, 66, 26, 61, 50, 85, 18},
 {36, 4, 65, 52, 41, 84, 28, 76, 17, 60}, {35, 4, 83, 59, 27, 51, 75, 19, 43, 67},
 {34, 4, 47, 72, 80, 58, 15, 39, 23, 91}, {33, 4, 38, 82, 49, 14, 90, 71, 57, 25},
 {32, 4, 56, 89, 16, 73, 40, 24, 81, 48}, {39, 29, 63, 50, 71, 76, 88, 24, 19, 10},
 {48, 29, 44, 60, 77, 91, 67, 18, 9, 25}, {57, 29, 55, 43, 85, 69, 81, 8, 23, 17},
 {66, 29, 87, 80, 27, 16, 7, 41, 49, 64}, {75, 29, 68, 90, 15, 6, 28, 53, 61, 40},
 {84, 29, 79, 73, 5, 26, 14, 58, 45, 51}, {41, 30, 70, 24, 53, 18, 82, 58, 8, 83},
 {51, 30, 86, 25, 72, 64, 17, 10, 74, 40}, {61, 30, 78, 23, 19, 89, 45, 65, 49, 9},
 {71, 30, 46, 27, 85, 5, 56, 77, 15, 52}, {81, 30, 62, 28, 7, 47, 67, 42, 88, 14},
 {91, 30, 54, 26, 38, 76, 6, 16, 59, 69}, {77, 21, 35, 58, 42, 10, 90, 16, 65, 55},
 {69, 21, 37, 40, 80, 52, 9, 83, 63, 14}, {88, 21, 36, 49, 8, 73, 59, 44, 15, 74},
 {53, 21, 32, 85, 19, 38, 79, 72, 7, 60}, {45, 21, 34, 67, 56, 87, 17, 50, 82, 6},
 {64, 21, 33, 76, 68, 18, 47, 5, 43, 89}, {72, 31, 57, 16, 50, 83, 28, 5, 44, 78},
 {89, 31, 39, 14, 27, 60, 74, 55, 70, 6}, {82, 31, 48, 15, 65, 26, 43, 86, 7, 63},
 {42, 31, 84, 19, 8, 68, 54, 80, 56, 25}, {59, 31, 66, 17, 79, 46, 9, 24, 90, 47},
 {52, 31, 75, 18, 62, 10, 87, 38, 23, 73}, {80, 12, 61, 5, 24, 55, 67, 38, 86, 36},
 {73, 12, 41, 6, 85, 25, 63, 78, 35, 47}, {90, 12, 51, 7, 44, 76, 23, 37, 56, 70},
 {50, 12, 91, 8, 65, 62, 33, 27, 79, 40}, {43, 12, 71, 9, 54, 32, 74, 58, 28, 87},
 {60, 12, 81, 10, 34, 83, 46, 68, 49, 26}, {27, 3, 88, 38, 58, 78, 17, 68, 37, 48},
 {26, 3, 77, 47, 19, 40, 70, 36, 57, 87}, {28, 3, 69, 56, 86, 18, 49, 39, 79, 35},
 {24, 3, 64, 65, 44, 34, 75, 85, 54, 14}, {23, 3, 53, 74, 33, 84, 63, 16, 67, 46},
 {25, 3, 45, 83, 66, 55, 32, 62, 15, 76}, {85, 20, 82, 68, 16, 36, 9, 62, 39, 51},
 {76, 20, 72, 87, 8, 14, 35, 48, 61, 46}, {67, 20, 89, 79, 37, 10, 15, 41, 54, 57},
 {58, 20, 52, 44, 86, 66, 81, 19, 33, 6}, {49, 20, 42, 63, 75, 91, 70, 5, 17, 32},
 {40, 20, 59, 55, 71, 78, 84, 34, 7, 18}, {19, 22, 90, 62, 5, 48, 74, 34, 41, 69},
 {18, 22, 80, 46, 65, 6, 57, 51, 88, 32}, {17, 22, 73, 54, 39, 83, 7, 77, 33, 61},
 {16, 22, 60, 86, 37, 71, 45, 8, 75, 47}, {15, 22, 50, 70, 81, 64, 35, 38, 9, 84},
 {14, 22, 43, 78, 53, 36, 91, 66, 56, 10}}
```

Algebarske strukture od kojih se konstruiraju ravnine

Konačno polje reda četiri:

+	0	1	2	3
0	0	1	2	3
1	1	0	3	2
2	2	3	0	1
3	3	2	1	0

·	0	1	2	3
0	0	0	0	0
1	0	1	2	3
2	0	2	3	1
3	0	3	1	2

Hallovo skoropolje reda devet:

+	0	1	2	3	4	5	6	7	8
0	0	1	2	3	4	5	6	7	8
1	1	2	0	4	5	3	7	8	6
2	2	0	1	5	3	4	8	6	7
3	3	4	5	6	7	8	0	1	2
4	4	5	3	7	8	6	1	2	0
5	5	3	4	8	6	7	2	0	1
6	6	7	8	0	1	2	3	4	5
7	7	8	6	1	2	0	4	5	3
8	8	6	7	2	0	1	5	3	4

·	0	1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8
2	0	2	1	6	8	7	3	5	4
3	0	3	6	2	5	8	1	4	7
4	0	4	8	7	2	3	5	6	1
5	0	5	7	4	6	2	8	1	3
6	0	6	3	1	7	4	2	8	5
7	0	7	5	8	3	1	4	2	6
8	0	8	4	5	1	6	7	3	2

Andreovo kvazipolje reda 16:

+	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
0	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	1	0	3	2	5	4	7	6	9	8	11	10	13	12	15	14
2	2	3	0	1	6	7	4	5	10	11	8	9	14	15	12	13
3	3	2	1	0	7	6	5	4	11	10	9	8	15	14	13	12
4	4	5	6	7	0	1	2	3	12	13	14	15	8	9	10	11
5	5	4	7	6	1	0	3	2	13	12	15	14	9	8	11	10
6	6	7	4	5	2	3	0	1	14	15	12	13	10	11	8	9
7	7	6	5	4	3	2	1	0	15	14	13	12	11	10	9	8
8	8	9	10	11	12	13	14	15	0	1	2	3	4	5	6	7
9	9	8	11	10	13	12	15	14	1	0	3	2	5	4	7	6
10	10	11	8	9	14	15	12	13	2	3	0	1	6	7	4	5
11	11	10	9	8	15	14	13	12	3	2	1	0	7	6	5	4
12	12	13	14	15	8	9	10	11	4	5	6	7	0	1	2	3
13	13	12	15	14	9	8	11	10	5	4	7	6	1	0	3	2
14	14	15	12	13	10	11	8	9	6	7	4	5	2	3	0	1
15	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0

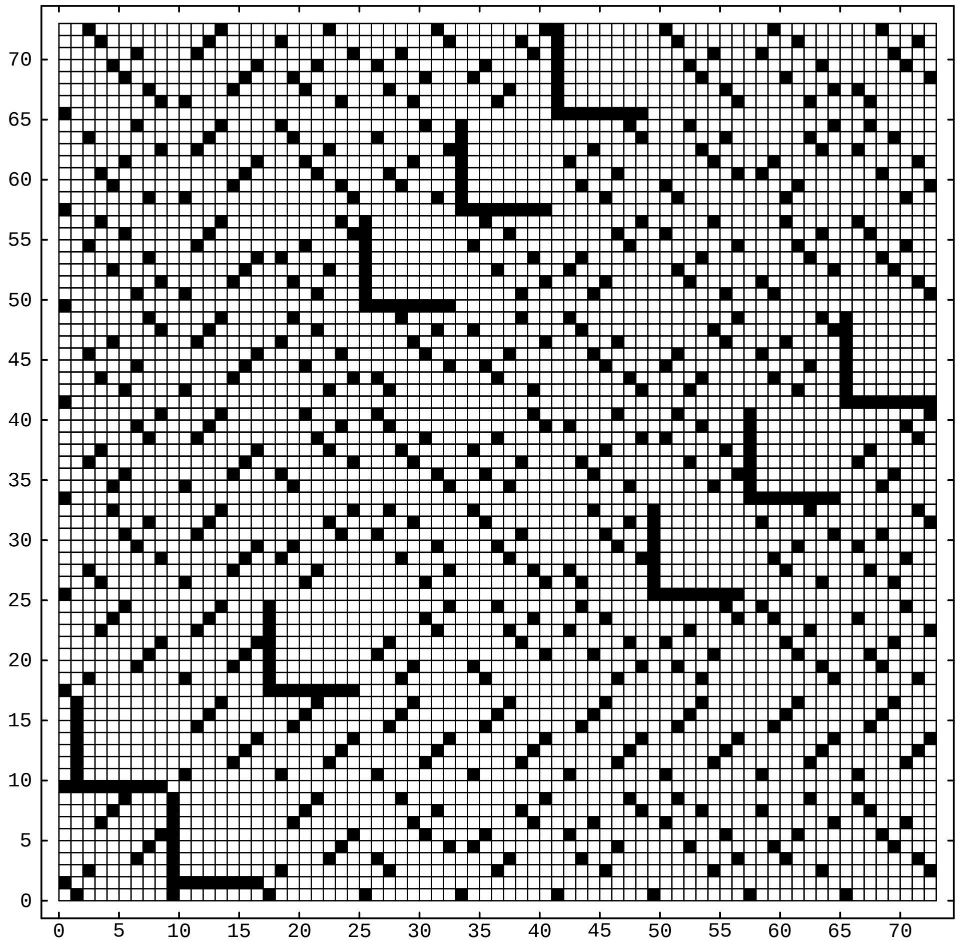
·	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
2	0	2	4	6	8	10	12	14	3	1	7	5	11	9	15	13
3	0	3	6	5	12	15	10	9	11	8	13	14	7	4	1	2
4	0	4	12	8	7	3	11	15	9	13	5	1	14	10	2	6
5	0	5	15	10	2	7	13	8	6	3	9	12	4	1	11	14
6	0	6	10	12	13	11	7	1	4	2	14	8	9	15	3	5
7	0	7	14	9	15	8	1	6	13	10	3	4	2	5	12	11
8	0	8	3	11	6	14	5	13	12	4	15	7	10	2	9	1
9	0	9	8	1	11	2	3	10	14	7	6	15	5	12	13	4
10	0	10	7	13	14	4	9	3	15	5	8	2	1	11	6	12
11	0	11	5	14	10	1	15	4	7	12	2	9	13	6	8	3
12	0	12	11	7	5	9	14	2	10	6	1	13	15	3	4	8
13	0	13	9	4	1	12	8	5	2	15	11	6	3	14	10	7
14	0	14	1	15	3	13	2	12	5	11	4	10	6	8	7	9
15	0	15	13	2	9	6	4	11	1	14	12	3	8	7	5	10

Incidencijske matrice konačnih ravnina

Projektivna ravnina reda tri:

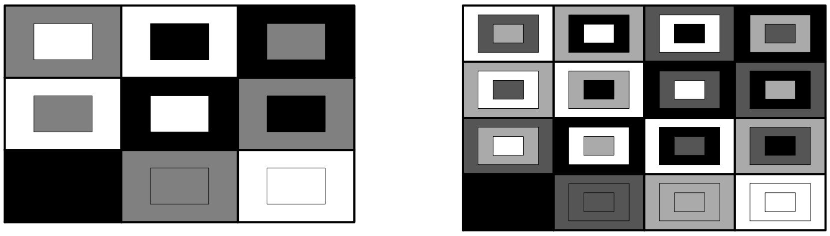
0	1	0	0	1	0	0	1	0	0	1	0	0
1	0	0	0	1	1	1	0	0	0	0	0	0
0	0	0	1	1	0	0	0	0	1	0	1	0
0	0	1	0	1	0	0	0	1	0	0	0	1
1	1	1	1	0	0	0	0	0	0	0	0	0
0	1	0	0	0	0	1	0	0	1	0	0	1
0	1	0	0	0	1	0	0	1	0	0	1	0
1	0	0	0	0	0	0	0	0	0	1	1	1
0	0	0	1	0	0	1	0	1	0	1	0	0
0	0	1	0	0	1	0	0	0	1	1	0	0
1	0	0	0	0	0	0	1	1	1	0	0	0
0	0	1	0	0	0	1	1	0	0	0	1	0
0	0	0	1	0	1	0	1	0	0	0	0	1

Projektivna ravnina reda 8:



Potpuni skupovi međusobno ortogonalinih latinskih kvadrata

Reda tri i reda četiri (grafički prikaz):



Reda sedam:

1	2	3	4	5	6	7
2	3	4	5	6	7	1
3	4	5	6	7	1	2
4	5	6	7	1	2	3
5	6	7	1	2	3	4
6	7	1	2	3	4	5
7	1	2	3	4	5	6

1	2	3	4	5	6	7
3	4	5	6	7	1	2
5	6	7	1	2	3	4
7	1	2	3	4	5	6
2	3	4	5	6	7	1
4	5	6	7	1	2	3
6	7	1	2	3	4	5

1	2	3	4	5	6	7
4	5	6	7	1	2	3
7	1	2	3	4	5	6
3	4	5	6	7	1	2
6	7	1	2	3	4	5
2	3	4	5	6	7	1
5	6	7	1	2	3	4

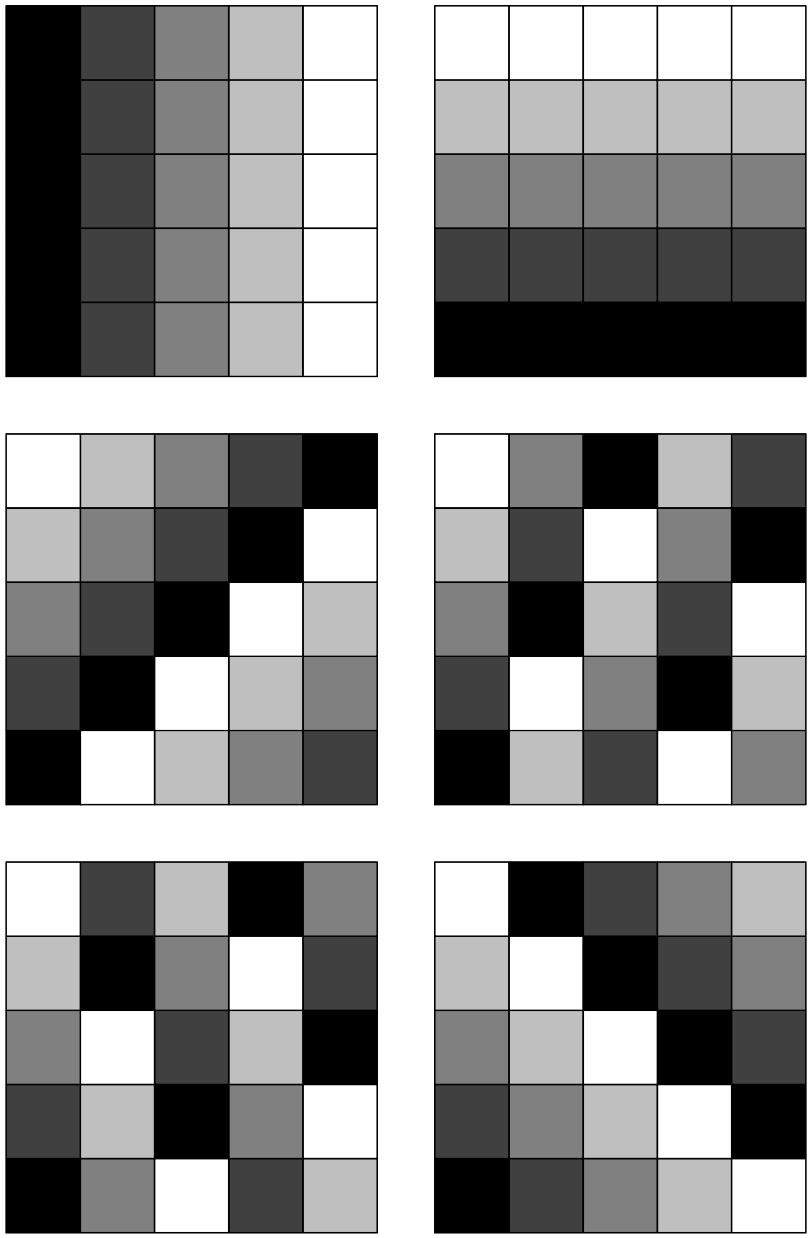
1	2	3	4	5	6	7
5	6	7	1	2	3	4
2	3	4	5	6	7	1
6	7	1	2	3	4	5
3	4	5	6	7	1	2
7	1	2	3	4	5	6
4	5	6	7	1	2	3

1	2	3	4	5	6	7
6	7	1	2	3	4	5
4	5	6	7	1	2	3
2	3	4	5	6	7	1
7	1	2	3	4	5	6
5	6	7	1	2	3	4
3	4	5	6	7	1	2

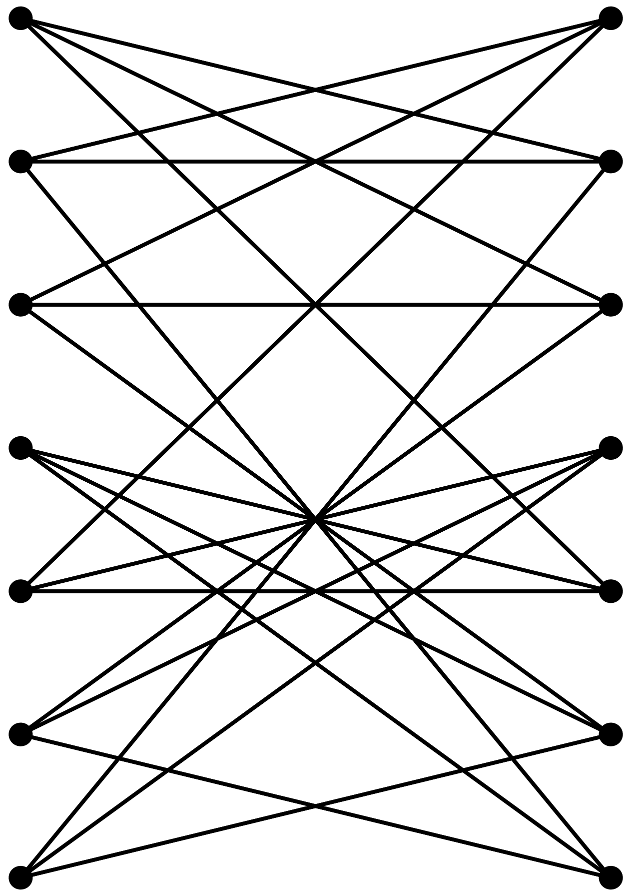
1	2	3	4	5	6	7
7	1	2	3	4	5	6
6	7	1	2	3	4	5
5	6	7	1	2	3	4
4	5	6	7	1	2	3
3	4	5	6	7	1	2
2	3	4	5	6	7	1

Vizualizacija afinih ravnina

Slika afine ravnine reda pet. Šest velikih kvadrata predstavljaju klase međusobno paralelnih pravaca. Unutar svakog od njih točke koje pripadaju jednom pravcu obojane su istom bojom (tj. nijansom sive).



Prikaz ravnina pomoću grafova



Projektivna ravnina reda dva.