# MATEMATIKA (BIOLOZI) – 1. KOLOKVIJ – RJEŠENJA

1. $$AB = \begin{bmatrix} 2x-y & -x \\ 2x+3y & x \end{bmatrix}, \quad BA = \begin{bmatrix} 2x & -2 \\ x^2+2yx & x-y \end{bmatrix}$$

$AB = BA$ ako i samo ako je $x = 2,\ y = 0$.

2. Matrica sustava je ekvivalentna matrici

$$\left(\begin{array}{ccc|c} 1 & 2 & 3-a & 1 \\ 0 & 1 & 5-a & 3 \\ 0 & 0 & a^2-2a & -a \end{array}\right)$$

Ako je $a = 2$, onda sustav nema rješenja.

Ako je $a = 0$, onda ima 1-parametarsko rješenje

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -5+7t \\ 3-5t \\ t \end{pmatrix}, \quad t \in \mathbb{R}.$$

Ako je $a \notin \{0, 2\}$, sustav ima jedinstveno rješenje

$$z = \frac{1}{2-a}, \quad y = \frac{1-2a}{2-a}, \quad x = \frac{4a-3}{2-a}.$$

3. $$A \sim \begin{pmatrix} 1 & -1 & 1 & 3 \\ 0 & 5 & 5 & -5 \\ 0 & 2 & 2 & -2 \\ 0 & 1 & 1 & -1 \end{pmatrix} \Longrightarrow r(A) = 2.$$

4. $$\left|\begin{array}{ccc} 1+x & 1 & 1 \\ 1 & 1+x & 1 \\ 1 & 1 & 1+x \end{array}\right| \begin{array}{cc} 1+x & 1 \\ 1 & 1+x \\ 1 & 1 \end{array} = (1+x)^3 + 1 + 1 - (1+x) - (1+x) - (1+x)$$

$$= 1 + 3x + 3x^2 + x^3 + 2 - 3 - 3x$$

$$= x^3 + 3x^2$$

Matrica je singularna ako je $x^3 + 3x^2 = 0$,

tj. ako je $x = 0$ ili $x = -3$.

5. $A^3 = \begin{pmatrix} -4 & 4 \\ 4 & -4 \end{pmatrix}$, $A^3 - 2I = \begin{pmatrix} -6 & 4 \\ 4 & -6 \end{pmatrix}$

$\det(A^3 - 2I) = 36 - 16 = 20$

$5 = \det B = \det(A^3C - 2C) = \det((A^3 - 2I) \cdot C)$

$= \det(A^3 - 2I) \cdot \det C = 20 \cdot \det C$

Binet-Cauchy

$\Rightarrow \det C = \frac{1}{4}$

$\Rightarrow \det(C^{-2}) = \frac{1}{(\det C)^2} = 16$,

6. $A(A^{-1} + X^{-1}) + (XA)^{-1} = 2I$

$I + AX^{-1} + \cancel{A}^{-1}X^{-1} = 2I$

$(A + A^{-1})X^{-1} = I \quad / \cdot X$

$\boxed{A + A^{-1} = X}$

$A^{-1} = \begin{pmatrix} 1 & -1 & 0 \\ 0 & -1 & 1 \\ -1 & 2 & 0 \end{pmatrix}$ (RAČUNAMO STANDARDNIM POSTUPKOM)

$X = A + A^{-1} = \begin{pmatrix} 3 & -1 & 1 \\ 1 & -1 & 2 \\ 0 & 3 & 1 \end{pmatrix}$

7. $\vec{a} \cdot \vec{b} = (2\vec{p} - 3\vec{q}) \cdot (\vec{p} + 5\vec{q}) =$

$= 2|\vec{p}|^2 + 10\vec{p} \cdot \vec{q} - 3\vec{p} \cdot \vec{q} - 15|\vec{q}|^2$

$= 2 \cdot 2^2 + 7 \cdot \vec{p} \cdot \vec{q} - 15 \cdot 1^2$

$= 8 + 7|\vec{p}| \cdot |\vec{q}| \cos\frac{\pi}{3} - 15 =$

$= -7 + 7 \cdot 2 \cdot 1 \cdot \frac{1}{2} = 0$.

$\Rightarrow \boxed{\angle(\vec{a}, \vec{b}) = 90^\circ}$

8. $\vec{AB} = B - A = (1, -2, -4)$

$\vec{AC} = C - A = (0, 1, 2)$

$$\vec{AB} \times \vec{AC} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & -2 & -4 \\ 0 & 1 & 2 \end{vmatrix} = 0 \cdot \vec{i} - 2 \cdot \vec{j} + 1 \cdot \vec{k}$$

$$P_{\Delta ABC} = \frac{1}{2} |\vec{AB} \times \vec{AC}| = \frac{1}{2} \sqrt{(-2)^2 + 1^2} = \frac{\sqrt{5}}{2}$$

9. $\frac{x-1}{2} = \frac{y-2}{-1} = \frac{z}{3} = t$

$x = 2t + 1$

$y = -t + 2$

$z = 3t$

$x + y + z = 11$

$2t + 1 - t + 2 + 3t = 11$

$\boxed{t = 2}$

$x = 5, \ y = 0, \ z = 6$, tj. $T(5, 0, 6)$

$\vec{s} = \vec{ST} = T - S = (2, 0, 4)$

tražena jednadžba pravca glasi

$$\frac{x-5}{2} = \frac{y}{0} = \frac{z-6}{4}$$