AMSC 2001

Ti e Table

1. Monday (ponedeljak)

8:30–9:15 Marcus Sarkis: New Coarse Spaces and Schwarz Method with Har onic Overlaps

9:15–9:30 Coffee (kava)

9:30–9:50 Senka Vuković and Luka Sopta: High-Order ENO and WENO Sche es with Flux Gradient and Source Ter Balancing

9:50–10:10 Nataša Bilić: Identification of discontinuous para eter in Navier-Stokes equations

10:10–10:50 Kreši ir Veselić: On the convergence of the Newton iteration

10:50–12:15 Marcus Sarkis: Schwarz Methods for Partial Differential Equations

17:00–17:45 Jesse Barlow: Least Squares, Total Least Squares and Co plete Orthogonal Deco positions

17:45–18:00 Coffee (kava)

18:00–18:20 Froilán M. Dopico: Co puting accurate eigenvalues and eigenvectors of sy - etric quasi-Cauchy atrices

18:20–18:40 Sanja Singer and Saša Singer: Factorization of Structured Sy etric Matrices

18:40–19:00 Ninoslav Truhar and Kreši ir Veselić: Opti ising the solution of a Ljapunov equation

19:00–19:20 Vjeran Hari: Convergence of Generalized Jacobi-type Methods

19:20–19:40 Tho as Benesch: A odel for recruit ent

2. Tuesday (utorak)

8:30–9:15 Rui Ralha: Accurate co putation of the SVD with one–sided ortogonal transfor ations

9:15–9:30 Coffee (kava)

9:30–9:50 Froilán M. Dopico and Julio Moro: On derivatives of orthonor al bases of invariant subspaces of Her itian atrices

9:50–10:10 Ivan Slapničar and Ninoslav Truhar: Relative Perturbation Theory for Hyperbolic Singular Value Deco position

10:10–10:30 Ivica Nakić and Kreši ir Veselić: Wielandt and Ky–Fan theore for atrix pairs

10:45–12:15 Marcus Sarkis: Schwarz Methods for Partial Differential Equations

Lunch (ručak)

17:00–17:20 Marko Vrdoljak: On principal eigenvalue of stationary diffusion proble with nonsy etric coefficients

17:20–17:40 Jan Rosenzweig: Mesoscale odels of poly eric liquids

17:40–18:00 Ivo Veselić: Regularity properties of the integrated density states of rando Schrödinger operators

18:00–18:15 Coffee

18:15–18:35 Andrija Raguž: Relaxation of so e energy functionals related to for ation of icrostructure

18:35–19:00 Josip Ta bača: A odel of irregular curved rods

19:00–19:20 Mladen Jurak: Averaging singularly perturbed convection-diffusion equation with non periodic velocity

19:20–19:40 Iryna Bosenko: Lagrangian relaxation of quadratic assign ent

3. Wednesday (srijeda)

9:00–11:30 Marcus Sarkis: Schwarz Methods for Partial Differential Equations

14:00–23:59 Excursion and dinner (izlet i večera)

4. Thursday

9:10–9:30 Knoll Peter: Application of an $\mu + \lambda$ evolutionary algorith to reconstruct single photon e ission co puted to ography (spect)–data.

9:30–9:50 Nelida Črnjarić–Žic, Senka Vuković and Luka Sopta: Nu erical approxi a-tion of the one–di ensional sedi ent transport equation

9:50–10:10 Ivo Beroš and Miljenko Marušić: Order of convergence for collocation with high order tension splines

10:10–10:30 Nela Bosner: Nu erical stability of Krylov subspace iterative ethods for solving linear syste s

10:30–13:00 Discussion on so e ongoing CFD projects in Croatia.

Lunch (ručak).

17:00–17:20 Božo Vrdoljak and Al a O erspahić: Qualitative Analysis of so e Solutions of Syste of Quasilinear Differential Equations

17:20–17:40 Martin Lazar: Microlocal energy density for hyperbolic syste s

17:40–18:00 To islav Marošević: On using L_p-nor s in esti ating distances

18:00–18:15 Coffee

18:15–18:35 Katalin Balla and Vil oš Horvat: Approxi ate solutions to so e second order linear recurrences

18:35–18:55 Saša Singer and Božidar Liščić: Calculation of Newton's Coefficient in the Law of Cooling

18:55–19:15 Mladen Rogina and Vanja Cecić: A Bezier type odel of Adriatic coast based on knot insertion and data reduction

19:15–19:35 Nenad Kranjčević, Milenko Stegić and Nikola Vranković: Nonlinear Proble s in Dyna ics by the Finite Ele ent Method in Ti e Do ain

5. Friday (petak)

- Discussions. The eeting adjourns.

Least Squares, Total Least Squares and Complete Orthogonal Decompositions

Jesse L. Barlow

Abstract. The singular value deco position (SVD) is the key atrix factorization necessary for understanding the solution of least squares proble s, total least squares proble s, and ost recently, latent se antic indexing (LSI). The ain drawback of the SVD is that it gives us far ore infor ation than any of these proble s needs. Usually, we just need the subspaces associated with "large" and "s all" singular values and good approxi ations will do.

Since the late 1960's, co plete orthogonal deco position (COD) has been used to obtain approxi ations to subspaces associated with "large" and "s all" singular values. If we only need this infor ation for one atrix, CODs offer only odest savings over the SVD. However, recursive least squares (LS) and total least squares (TLS) proble s require us to add or delete rows of data at each ti e step. For the SVD, present stable software requires $O(n^3)$ flops, but CODs can be odified in $O(n^2)$ flops and preserve their approxi ations of the necessary subspaces.

Algorith ic issues re ain in the COD approach to recursive LS and TLS proble s re ain, including condition and nor esti ation, choice of rank–one change algorith s, orthogonality issues, and deflation techniques. This talk will discuss our general approach to these issues.

Jesse L. Barlow, Co puter Science and Engineering 312 Pond Laboratory The Pennsylvania State University University Park, PA 16802–6106

A Model for Recruitment

Tho as Benesch

Abstract. The Model contains of 5 steps:

1. Represent: The broad business ilieu of the future is being cleared up, in which the top perfor er will be active.
2. Reclai : The valuable tasks and the future work services of the top perfor er are being elaborated.
3. Restrict: The criteria for the qualifications catalogue is being acquired. The detailed description of a different qualification criteria occurs. In addition, 4 to 5 levels of capability are being defined, beginning with the just accepted qualification to the ideal qualification.

4. Reward: The develop ent of a future pay ent odel. An orientation of the odel is indispensable both fro the e ployees achieve ent and qualification for a salary based on perfor ance.

5. Recruit: The top perfor er is finally being no inated, in other words, recruited.

With the Top-down top perfor er concept, anagers of rank ust take on the roll as first contact person for the e ployees regarding staff atters. The chance is growing for staff leaders to broaden their area of responsibility using the concepts fro the top perfor er odel and the bound develop ent of achieve ent that goes along with it.

The next step Regard should follow, after the top-perfor er is recruited. Regard: Syste atically develop ent of the qualification of the top perfor er.

Thomas Benesch, Fachhochschule Joanneu Infor ation Manage ent / General Manage ent and Organisation Alte Poststrae 149 8020 Graz Austria
phone: ++43/316/5453-8520
fax: ++43/316/5453-8501
e– ail: thomas.benesch@fh-joanneum.at

Order of Convergence for Collocation with Higher Order Tension Splines

Ivo Beroš and Miljenko Marušić

Abstract. Tension spline of order k is a function that, for given partition $x_0 < x_1 < \ldots < x_n$, on each interval $[x_i, x_{i+1}]$ satisfies differential equation $(D^k - \rho_i^2 D^{k-2})u = 0$, where ρ_i's are prescribed nonnegative real nu bers.

Many articles deal with tension splines of order four applied to collocation ethod for solving singularly perturbed boundary value proble

$$(\mathcal{L}u)(x) = \varepsilon u''(x) + b(x)u'(x) + c(x)u(x) = f(x), \quad (0 \le x \le 1)$$

with boundary conditions

$$u(0) = u_0, \quad u(1) = u_1\,.$$

Accuracy of considered approxi ations is $\mathcal{O}(h)$ or $\mathcal{O}(h^2)$ for s all perturbation para eter ε, depending on the choice of collocation points.

Here we present an algorith for collocation ethod with higher order tension splines, prove its convergence and obtain order of accuracy for the solution of singularly perturbed differential equation.

Ivo Beroš, Depart ent of Mathe atics, University of Zagreb, Bijenička cesta 30, 10000 Zagreb, Croatia.

e– ail: ivo.beros@math.hr

Miljenko Marušić, Depart ent of Mathe atics, University of Zagreb, Bijenička cesta 30, 10000 Zagreb, Croatia.
e– ail: miljenko.marusic@math.hr

Identification of discontinuous parameter in Navier-Stokes equations

Nataša Bilić

Abstract. A proble of identifying possibly discontinuous viscosity coefficient in Navier-Stokes equations is consider. General theore on existence of inverse proble and Galerkin approxi ations are proved in L^1-setting. A double discretization ethod with the variations constraints is used in two and three di ensional inverse proble .

Nataša Bilić, Faculty of Civil Engineering, Matice hrvatske 15, Split, Croatia

Numerical stability of Krylov subspace iterative methods for solving linear systems

Nela Bosner

Abstract. Very often used ethods for solving linear syste s are Krylov subspace iterative ethods. The usual stopping criterion for these ethods, is the o ent when so e nor of residual reaches tolerable value. Since all co putations are done in finite precision arith etic, we can check only so e approxi ation of the residual nor . The ain goal of our research is, to give esti ation for the real residual nor , calculated fro this approxi ation, in order to ake stopping criterion ore reliable. We have done this for two types of algorith s.

Nela Bosner, Depart ent of Mathe atica University of Zagreb, Bijenička cesta 30, 10000 Zagreb
e– ail: nela@math.hr

Numerical Linear Algebra

Bosenko I, Ohaka S.E, Peter Pee an.

Abstract. In recent years, a unified theory to a new approach to the quadratic assign ent based on the convex quadratic progra ing relaxations was perfor ed.

Many engineering proble s can be odelled as the ini ization (or axi ization) of a quadratic function subject to quadratic constraints. These odels are non convex in general; so finding a global solution is itself a very hard proble . We look at Lagrangian relaxations for these proble s. The Lagrangian relaxations are equivalent to se i definite progra ing relaxations, i.e. to the ini ization of a linear function of sy etric atrices. The i ple entation for the first ti e solved to so e extent several large scale proble s including nug30 and tho30 proble s. The co putations associated with these proble s are a ong the largest ever perfor ed in solving engineering proble s nu erically.

Bosenko I, Engineering Dept. The International Slovanic University Kharkov Ukraine
e– ail: nauka2001@ukr.net

Numerical Approximations of the One-Dimensional Sediment Transport Equations

Nelida Črnjarić-Žic, Senka Vuković and Luka Sopta

Abstract. The ain goal of our work is to investigate and co pare different athe atical for ulations and nu erical approxi ations for the one-di ensional bed-load sedi ent transport proble . We follow the work of Hudson and Sweby who described few different for ulations describing the physical pheno ena of one-di ensional bed-load sedi ent transport in channels and rivers and used experi entally obtained laws for sedi ent flux proposed by van Rijn. The resulting athe atical odel consists of three equations. Two of the represent conservation laws for one-di ensional shallow water equations, and the third is the conservation law governing bed-load sedi ent transport.

We concentrate on the ho ogeneous and non-ho ogeneous for ulation of the conservation laws and on the application of the 1st order Q-sche e (Ber udez, Vzquez), 2nd order Hubbard sche e with flux li iters and high-order ENO and WENO with the source ter deco posed (developed by the authors of this paper). We discuss and co pare nu erical results obtained on standard sedi ent transport test proble s.

Nelida Črnjarić-Žic, University of Rijeka, 51000 Rijeka, Vukovarska 58, Croatia
e– ail: nelida@rijeka.riteh.hr

Senka Vuković, University of Rijeka, 51000 Rijeka, Vukovarska 58, Croatia
e– ail: senka.vukovic@ri.hinet.hr

Luka Sopta, University of Rijeka, 51000 Rijeka, Vukovarska 58, Croatia
e– ail: sopta@rijeka.riteh.hr

Evolutionary Stability Analysis of Tree Based Decision Principles

A. Ekárt and S. Z. Né eth

Abstract. In ulticriteria decision proble s any subjective decisions ust be ade, such as the i portance of the different criteria and the values of the alternatives with respect to subjective criteria. Since subjective decisions are approxi ate, it is very i portant to analyze the sensitivity of results when s all odifications of the subjective values are ade.

When solving a ulticriteria decision proble , it is desirable to choose a decision principle that conducts to a solution as stable as possible. We consider a class of decision principles that are constructed fro ele entary decision principles and we perfor the sensitivity analysis of these decision principles with respect to weights and alternative values. In both cases we introduce a global stability index, which easures the stability of the order of alternatives with respect to s all perturbations of alternative values and weights, respectively.

We then propose a ethod based on genetic progra ing that produces better decision principles than the co only used ones. New decision principles of the defined class are evolved by genetic progra ing where the driving force of evolution is stability. The theoretical expectations are validated by two case studies.

References

[1] Wolfgang Banzhaf, Peter Nordin, Robert E. Keller and Frank D. Francone, *Genetic Programming: An Introduction, Morgan Kaufmann, 1998*

[2] Péter Csáki, Ferenc Fölsz, Támas Rapcsák and Zoltán Sági; On tender evaluations, *Jurnal of Decision Systems, 7,179-194,(1998)*

[3] John R. Koza, Genetic Programming: On the Programing of Computers by Means of Natural Selection, *MIT Press, 1992*

[4] Cszaba Mészáros and Tamás Rapcsák; On sensitivity analysis for a class of decision systems, *Decision Support Systems, 16(3), 231-240,(1996).*

[5] Tomas L. Saaty; The analytic hierarchy process. Planing, priority setting, resource allocation, *McGraw-Hill, 1980.*

[6] L. Saaty and Luis G. Vargas, Uncertainty and Rank Order in the Analytic Hierarchy Process, *European J. Oper. Res.,32(1),107-117,(1987).*

A. Ekárt, Co puter and Auto ation Research Institute, Hungarian Acade y of Sciences, 1518 Budapest, P.O.B. 63, Hungary
e– ail: ekartsztaki.hu

S. Z. Németh, Co puter and Auto ation Research Institute, Hungarian Acade y of Sciences, 1518 Budapest, P.O.B. 63, Hungary
e– ail: snemethsztaki.hu

Computing accurate eigenvalues and eigenvectors of symmetric quasi-Cauchy matrices.

Froilán M. Dopico

Quasi-Cauchy atrices generalize Cauchy atrices via ultiplication by diagonal atrices on both sides. These atrices appear frequently in the literature as test atrices and also in applications like rational interpolation proble s. De el has shown in [1] that it is possible to co pute singular value deco positions of quasi-Cauchy with high relative accuracy. We extend these results and give two algorith s to co pute spectral deco positions of sy etric quasi-Cauchy atrices with high relative accuracy. The first algorith consists just in assigning the correct signs to the singular values co puted by De el's algorith . Reference [2] has proved that this can be accurately done. The second algorith begins by co puting an accurate enough $G\,J\,G^T$ ($J = I_p \oplus (-I_{n-p})$) factorization of the quasi-Cauchy atrix, adapting an algorith in [1] to co pute accurate co ponentwise Schur co ple ents of quasi-Cauchy atrices. Then the J-orthogonal Jacobi-like algorith of Veselić and Slapničar [4], [3] is applied to the atrices G, J. Theory and nu erical experi ents of these algorith s are presented.

References

[1] J. Demmel, *Accurate singular value decompositions of structured matrices*, SIAM J. Matrix Anal. Appl., 21 (1999), pp. 562-580.

[2] F. M. Dopico, J. M. Molera, J. Moro, *An orthogonal high relative accuracy algorithm for the symmetric eigenproblem*, in preparation.

[3] I. Slapničar, *Accurate Symmetric Eigenreduction by a Jacobi Method*, PhD Thesis, Fachbereich Mathematik Fernuniversität, Gesamthochschule Hagen, Germany, 1992.

[4] K. Veselić, *A Jacobi eigenreduction algorithm for definite matrix pairs*, Numer. Math., 64 (1993), pp. 241-269.

Froilán M. Dopico, Departa ento de Mate áticas, Universidad Carlos III de Madrid, 28911-Leganés, Spain
e– ail: dopico@math.uc3m.es
phone: 341 624 9446

Stabilization of Process of Random Walks in Graphs

Gerasin S. N., Ivaschenko V.V., Voskoboinic O. N.

Abstract. Modeling a lot of processes in econo ics, ecology, biology and techniques it is often easier to resort to the odels connected with rando walks in graphs. Walks in graphs are used in the research of rapidly evolving processes in econo ics in particular. The walks that beco e stabilize after a long period are of

peculiar interest. The last eans that after a very long period the basic characteristics of walk process ay be localizated in the vicinity of preset values.

There is considered a walk process in a graph that constitutes a tree in the report. Enu erating the vertexes of a graph walks in graph ay be reduced to the research of so e Markov's process. This process ay be considered as a discrete Markov's chain generated by Markov's process in incessant ti e. In application the case when process is nonunifor and has variable duration of conversions is of particular interest. An infinitesi al atrix $\Lambda(t) = (\lambda_{ij}(t))$ is used in researching of the process. Let's adduce the conditions that ay stabilize the process during a finite period if they are realized.

Suppose the sequence of utually disjoint periods $\{[s_k, t_k]\}_{k=1}^{\infty}, s_k < t_k < s_{k+1,} s_k \uparrow t_0 \leq \infty$ and the order of indexes $j_k, k = 1, 2, ...$ (j_k enu erates the states) for which

$$\sum_{k=1}^{\infty} \inf_i \left| \int_{s_k}^{t_k} \lambda_{ij_k}(s) ds \right| = \infty,$$

oreover in the set $[s_0, t_0) \setminus \cup_{k=1}^{\infty} [s_k, t_k)$ the nor of atrix $\Lambda(s)$ is bounded with the sa e constant. Suppose $\Lambda(t)$ to be continuous on $[s_k, t_k)$ and exist the li it

$$\lim_{k \to \infty} \vec{p}\ (\tau_k) = \vec{p},$$

where $\tau_k \in [s_k, t_k)$, $\vec{p}\ (\tau_k)$ - null eigenvector of the atrix $\Lambda(\tau_k)$. Then for any j and any initial probability distribution specified in the point $s \in [s_0, t_0]$

$$\lim_{t \uparrow t_0} p_j(s, t) = p_j.$$

Here p_j- j co ponent of the vector $\vec{p}$. If the series convergence but its su is quite big then t_0 is its point of stabilization.

Iterative-Collocation Methods with Corrections for Volterra-Fredholm Integral Equations

Lechoslaw Hącia

Abstract. We consider the following integral equation

$$u(x,t) = f(x,t) + \int_0^t \int_M K(x,t,y,s) u(y,s)\, dy\, ds,$$

where u is unknown function in D, $D = M \times [0,T]$, M is a closed subset of Euclidean space R^m and K is defined in $\Omega = \{(x,t,y,s) : 0 \leq s \leq t \leq T;\ x, y \in M\}$.

Volterra-Fredhol integral equations play very i portant role in the theory of parabolic initial-boundary value proble s (heat conduction proble s) and various physical, technological and biological proble s.

These equations arise in the athe atical odelling of the spatio-te poral develop ent of an epide ic. Spread of the disease in a given population can be described by Volterra-Fredhol equations.

In this paper we propose the iterative-collocation ethod

$$\begin{aligned} u_k(x,t) &= f(x,t) + \int_0^t \int_M K(x,t,y,s)[u_{k-1}(y,s) + p_k^n(y,s)]\,dy\,ds, \qquad k = 1,2,\ldots, \\ u_0(x,t) &= f(x,t) \end{aligned} \tag{1}$$

with corrections p_k^n based on the interpolation polyno ial of the Lagrange type with given collocation points.

Presented ethods lead to a syste of linear algebraic equations or a syste of Volterra integral equations.

The convergence is proved and an error esti ate is established. The considered theory is illustrated by nu erical exa ples.

References

[1] H. Brunner ,,On the numerical solution of nonlinear Volterra-Fredholm integral equations", SIAM J. Numer. Anal., 27, (1990), 987–1000.

[2] L. H?cia, ,,On approximate solutions for integral equations of the mixed type", Zeit. Ang. Math. Mech., 76, (1996), 415–416.

[3] J. P. Kauthen, ,,Continuous time-collocation methods for Volterra-Fredholm integral equations", Numer. Math., 56, (1989), 409–429.

Lechos?aw H?cia, Institute of Mathe atics, Pozna? University of Technology, Piotrowo 3A, 60–965 Pozna?, Poland
e– ail: lhacia@math.put.poznan.pl

Convergence of Generalized Jacobi-type Methods

Vjeran Hari

Abstract. A general for of Jacobi-type processes for solving atrix eigenvalue and singular value proble s is considered. This for includes the two extre es: "sequential" and "parallel" ethods. Pivot strategies are also generalized to suite the new fra e. Sufficient conditions for the convergence to diagonal for of such a general process is proved. These conditions can be viewed as generalized Forsythe-Henrici conditions. They can be applied equally well to si ple ethods such as standard Her itian Jacobi ethods, Kogbetliantz ethods for triangular or butterfly atrices and to ore co plex ethods such as the nor -reducing ones.

Vjeran Hari, University of Zagreb, Depart ent of Mathe atics, Bijenička cesta 30, 10002 Zagreb, Croatia
e– ail: hari@math.hr

Microlocal energy density for hyperbolic systems

Martin Lazar

Abstract. Starting fro the ethod for co puting icrolocal energy density, which was developed independently by Gérard, and Francfort and Murat, we want to co pute that very density for the hyperbolic syste

$$A^0\partial_0 v + \sum_1^d A^k \partial_k v + Bv = G.$$

The energy connected to the hyperbolic syste is given by the relation

$$E := \frac{1}{2}\langle A^0 v, v\rangle.$$

We want to express the energy li it of the sequence of initial proble s with the energy of initial conditions. The basic calculus tool are H- easures (also known as icrolocal defect easures). We associate an H- easure to the sequence of gradients of solutions to our syste and it represents the desired icrolocal energy density.

We have deter ined the equation satisfied by the corresponding H- easure. In the case of the constant coefficients it reduces to a hyperbolic syste si ilar to the initial one.

Rewriting the wave equation as a hyperbolic syste , we calculated the associated H- easure for the oscillating sequence of the initial conditions. The result is analogous to the one obtained by the direct calculus of H- easure fro the D'Ale bert's for ula for the solution of the wave equation.

This is a joint work with Nenad Antonić.

On using L_p-norms in estimating distances

To islav Marošević

Abstract. The proble of deter ining or esti ating the distances a ong the objects in space appears in any areas of application (e.g. road distances for continuous location odels, various distances in Geographical Infor ation Syste , etc., see in: *Facility Location, Survey of Application and Methods, Z. Drezner (Ed.), (Chapt. 1. Estimating Distances), Springer Series in Operations Research, Springer-Verlag New York, 1995*).

The so-called distance prediction function ay be used to esti ate the distance between the points with given coordinates. As a distance predicting function one can use the weighted L_2-nor and generally the weighted L_p-nor $(p \geq 1)$. In such weighted L_p-nor odels or τL_p odels, it is necessary to esti ate para eters

τ and p. In that proble we also look at so e special properties of L_p-nor s, particularly the so-called directional bias of nor s.

Tomislav Marošević, Depart ent of Mathe atics, University of Osijek, Trg L. Mirskog 1/II, HR-31000 Osijek, Croatia
e– ail: Tomislav.Marosevic@etfos.hr
fax: +385 (0)31 211-637

Derivation of a non-linear filtration law by homogenization

Eduard Marušić-Paloka

Abstract. We consider the stationary viscous inco pressible flow through a rigid porous ediu . For a periodic porous ediu with a period ε the ho ogenization ethod gives different results depending on the relationship between the Reynolds nu ber, the Freude's nu ber and ε. If both, the Reynolds nu ber and the inverse of the Freude's nu ber are of order ε^{-1} the filtration law is nonlocal and nonlinear. We call that, the critical case. In the subcritical case the fast and slow scales are well separated. We can find the sequence of linear auxiliary proble s and with their solutions we can for an asy ptotic expansion of the solution. In the critical case, we prove the existence of a unique solution for nonlinear ho ogenized proble . Moreover, we separate the fast fro the slow scale and we find a Taylor's expansion for the filtration velocity as a function of the pressure gradient. Finally, we prove the convergence of the ho ogenization process as well as the error esti ates. (Work in collaboration with A.Mikelić and A.Bourgeat)

Eduard Marušić-Paloka, Depart ent of Mathe atics, University of Zagreb, Bijenička 30, 10000 Zagreb, Croatia

On derivatives of orthonormal bases of invariant subspaces of Hermitian matrices

Froilán M. Dopico and Julio Moro

Abstract. Explicit for ulas are given for the derivatives of orthonor al bases of invariant subspaces of linear Her itian perturbations $A(t) = A+tE$ of a Her itian atrix A. A characterization is provided for all possible differentiable orthonor al bases via Sylvester equations, together with a procedure allowing to construct differentiable orthonor al bases starting fro arbitrary ones.

As a direct consequence of these results, derivatives of bases of left and right singular subspaces are obtained for arbitrary atrices using the Jordan-Wielandt for .

JULIO MORO, Departa ento de Mate áticas, Universidad Carlos III de Madrid, Avda. Universidad 30, 28911-Leganés (Spain)

Wielandt and Ky–Fan theorem for matrix pairs

Ivica Nakić and Kreši ir Veselić

Abstract. The classical Wielandt and Ky–Fan theore s are widely used in atrix theory. Using these results one can obtain various eigenvalue perturbation bounds.

The classical Wielandt theore states that

$$\sup_{\mathcal{M}_1\subset\mathcal{M}_k,\dim\mathcal{M}_j=i_j}\ \inf_{X=[x_1\ldots x_k],\ x_j\in\mathcal{M}_j\,X^*X=I_k}\ \sum_{j=1}^{k} X^*AX$$

equals the su $\lambda_{i_1}+\cdots+\lambda_{i_k}$, where $\lambda_1\geq\ldots\lambda_n$ are the eigenvalues of the Her itian atrix A, and $1\leq i_1<\ldots<i_k\leq n$, and the classical Ky–Fan theore states that

$$\inf_{X\in\mathbf{C}^{n\times p}X^*X=I_p}\mathrm{Tr}(\mathrm{X^*AX})$$

equals the su of the p s allest eigenvalues of the Her itian atrix $A\in\mathbf{C}^{n\times n}$.

So e generalizations of these theore s to the class of Her itian atrix pairs are already known. The authors obtained a generalizationa to a wider class of atrices. Especially, the definite atrix pairs satisfy our assu ptions.

For exa ple, the generalized Ky–Fan theore states that

$$\inf_{X\in\mathbf{C}^{N\times p}X^*BX=I_p}\mathrm{Tr}(\mathrm{X^*AX})$$

equals the su of the first p "good" eigenvalues of the positive type of the atrix pair (A,B).

Also, so e new eigenvalue perturbation esti ates will be presented. The application is ade on a class of partially overda ped quadratic atrix pencils.

IVICA NAKIĆ, Lehrgebiet Mathe atische Physik, Fernuniversität, 58084 Hagen Ger any, e- ail: ivica.nakic@fernuni-hagen.de (currently on leave fro University of Zagreb, Depart ent of Mathe atics, Zagreb, Croatia
e– ail: nakic@math.hr

KREŠIMIR VESELIĆ, Lehrgebiet Mathe atische Physik, Fernuniversität, 58084 Hagen Ger any
e– ail: kresimir.veselic@fernuni-hagen.de

Approximate solutions to some second order linear recurrences

Katalin Balla and Vil os Horvat

Abstract. The second order linear recurrence

$$y_{n+1} + b_n y_n + h_n y_{n-1} = e_n \tag{2}$$

with real coefficients was considered in an earlier paper assu ing that

$$\lim_{n\to\infty} b_n = b_\infty, \quad \lim_{n\to\infty} h_n = h_\infty \quad \text{and} \quad \lim_{n\to\infty} e_n = e_\infty$$

hold with constants $b_\infty, h_\infty, e_\infty$ such that the quadratic equation $q^2 + b_\infty q + h_\infty = 0$ has a pair of (real) roots q_1, q_2 such that $|q_1| > 1 > |q_2| > 0$. Under these conditions it was stated that a solution of (2) is bounded, i.e.

$$|y_n| = O(1), \quad n \to \infty \tag{3}$$

iff for any sufficiently large n it satisfies the first order recurrence

$$y_{n+1} = \omega_n y_n + \psi_n$$

where the pair ω and ψ is the solution of the proble

$$\begin{aligned} \omega_{n+1}\omega_n + b_{n+1}\omega_n + h_{n+1} &= 0, \qquad \omega_\infty := \lim_{n\to\infty} \omega_n = q_2 \\ \psi_{n+1} + (\omega_{n+1} + b_{n+1})\psi_n - e_{n+1} &= 0, \qquad \psi_\infty := \lim_{n\to\infty} \psi_n = -\frac{e_\infty}{q_1 - 1}. \end{aligned}$$

When the coefficients in (2) have asy ptotic series representation with respect to nonpositive powers of n, so have the functions ω and ψ.

The ai of this paper is to show that given an error bound ε, one can co pute a value $N(S, \varepsilon)$ such that for $n \geq N(S, \varepsilon)$ the approxi ation errors of partial su s $\tilde{\omega}_n$ and $\tilde{\psi}_n$ for ω_n and ψ_n with S ter s do not exceed ε .

Provided the proble (2), (3) is supplied with a left boundary condition

$$y_0 = \bar{y}_0, \tag{4}$$

the approxi ate boundary value proble consisting of (2), (4) and

$$y_{N+1} = \tilde{\omega}_N y_N + \tilde{\psi}_N$$

given on the finite interval $[0, N+1]$ furnishes us with an approxi ate solution of the exact boundary value proble (2), (3), (4) posed on the infinite interval $[0, \infty)$.

Katalin Balla, Co puter and Auto ation Research Institute Hungarian Acade y of Science, 1518 Budapest, P.O.Box 63. Hungary.

e– ail: balla@sztaki.hu

Vilmoš Horvat, Depart ent of Mathe atics, University of Zagreb, Bijenička cesta 30, 10000 Zagreb, Croatia.
e– ail: horvat@math.hr

Calculation of non-stationary temperature field in ultilayer under boundary conditions of the third sort

H. Hovhanissyan

Abstract. Is considered the unli ited plate with known initial distribution of te perature is. On borders between layers the ideal ther al contact is carried out (i.e. the boundary conditions of the 4-th sort are fulfiled). On external boundary surfaces the boundary conditions of the third sort are given.

For the description of a te perature field in ultilayer plate the syste of the differential equations of heat conductivity, written down in cartesian coordinates is sed. This syste is joining initial and boundary conditions, which are giving the co plete athe atical description of the task.

The syste of the equations with initial and boundary conditions is solved certainly by the ethod of finite differences.

As an exa ple the results of calculation of a non-stationary field in three-layered preparation of footwear consisting fro calf-leather of chro e tanning, poly-cloroprene glue-solution and leather lining. In the initial o ent of ti e all points of preparation of top of footwear have constant te perature. The leather lining enters the subsequent o ent of ti e in ther al contact with footstep of the an, and the calf-leather of chro e tanning with an environ ent (boundary conditions of the first sort).

Curvs of distributions of te peratures on thickness of reparation of top of footwear for the various o ents of ti e are obtained. The obtained results will be well coordinated with available experi ental and theoretical data.

H. Hovhanissyan, Northern Depart ent of NSSP, Leningradian st 5a, Gyu ri, Shirak Region, Ar enia, 377515

Qualitative Analysis of some Solutions of System of Quasilinear Differential Equations

Božo Vrdoljak ans Al a O erpašić

Abstract. In this paper we stady the existence and behaviour of so e solutions of the syste of quasilinear differential equations. The behaviour of solutions in

the neighbuorhood of an arbitrary curve is considered, with extraordinary attention on so e special cases. The obtained results contain an answer to the question on approxi ation as well as stability of solutions whose existence is established. The errors of the approxi ation are defined by the functions that can be sufficiently s all. The qualitative analysis theory of differential equations and the topological retraction ethod are used.

Božo Vrdoljak, Faculty of Civil Engineering, University of Split 21000 Split, Croatia

Alma Omerspahić, Faculty of Mechanical Engineering in Zenica, University of Sarajevo 72000 Zenica, Bosnia and Herzegovina

Relaxation of some energy functionals related to formation of microstructure

Andrija Raguž

Abstract. In this paper we investigate asy ptotic behaviour of certain energy functionals which appear in odelling of an- ade aterials having ability to perfor icrostructures. In particular, following [1], we study the for and the properties of the li iting relaxed energy associated with such a fa ily of functionals, including a high-oscillatory lower order ter s.

We will explain the physical background and the reasons why it is difficult to obtain a natural for of a relaxed functional at the li it.

Further ore, we prove so e preli inary results which assure that a li iting behaviour can esentally differ fro expected one, and we will propose so e new relaxations which are belived to be consistent with actual physical observations.

References

[1] Giovanni Alberti, Stefan Müller: in preparation

Andrija Raguz , Depart ent of Mathe atics, Zagreb, Croatia

Accurate computation of SVD with one-sided orthogonal transformations

Rui Ralha

Abstract. It has been known for quite so e years that one-sided Jacobi's ethod can be significantly ore accurate than QR for the co putation of singular values

of certain atrices. The price to pay for using Jacobi's ethod is, of course, the increase in the co putational co plexity.

In this lecture we will discuss a ethod which is based on the transfor ation of a atrix A into A_{n-2} which is such that $A_{n-2}^T A_{n-2}$ is tridiagonal. This is one-sided transfor ation since it involves only post ultiplication of A with Householder atrices, thus preserving high accuracy of the singular values when $A = DX$ where D is diagonal and X is uch better conditioned than A.

The way to proceed depends on how uch one needs to know about the SVD of the given A. If only singular values are sought then a si ple orthogonalization of adjacent colu ns of A_{n-2} delivers bidiagonal for to which the dqds ethod, or any alternative ethods, can be applied. The proposed transfor ation is quite straightforward but there are, however so e less obvious aspects that need to be taken into account to guarantee the accuracy of the co puted bidiagonal for and these will be addressed in the talk. Nu erical exa ples carried out with Matlab's i ple entation of our algorith will be presented to illustrate the ain points of the error analysis.

We will also outline the ain ideas behind another ethod which, like one-sided Jacobi's ethod, produces a atrix whose colu ns are sufficiently orthogonal to give not only accurate singular values but also accurate singular vectors.

A Bezier type model of Adriatic coast based on knot insertion and data reduction

Mladen Rogina and Vanja Cecić

Abstract. We describe athe atical techniques needed to develop a vector odel of a co plicated geographical ap, and apply those techniques to the odeling of the Adriatic coast. Cubic Bezier curves are used in the odel, and Oslo-type algorith s are used in the control polygon description. Opti ization questions based on data reduction software developed by Lyche *et all.* are considered. We discuss so e real-world proble s, like WGS-84 cartographic projections, GPS support and navigational proble s in the above setting. A feedback of this practical proble s to tangent-discontinuous para etric spline functions ay be of interest.

We present an elaborate software NAV2001, which can acco plish untrivial navigational tasks, such as deter ination of routes and positions, as well as adding sy bols and infor ation to the ap. Database software is e ployed in this context, containing detailed plans, historical and navigational infor ation of various type, not to ention GSM, eteorological and Internet support as a *sine qua non* of the odern civilization.

The software itself has been extensively tested in the real world situations, and we give a brief description of auto-pilot, echo-sounder and si ilar NMEA-standard devices used in navigation, and touch so e open proble s including radar equip-ent and associated athe atical proble s. Further infor ation and so e freely

downloadable software is available on a web site http://www.cvs.hr/Nav2001hr.ht .

Mladen Rogina, Dept. of Mathe atics, University of Zagreb, Bijenička 30, Zagreb, P.O.Box 10002, Croatia.
e– ail: rogina@math.hr
http://www.math.hr
fax: 385-1-4680335

Vanja Cecić, CV Siste i d.o.o., Padovčeva 3, Zagreb, P.O.Box 10000, Croatia.
e– ail: vanja@cvs.hr
http://www.cvs.hr
fax: 385-1-2395776

Mesoscale models of polymeric liquids

Jan Rosenzweig

Abstract. We shall outline two si ilar, although conceptually different co putational and theoretical esoscopic odels of poly eric liquids, that are capable of describing the icrostructures in ost flow conditions. In both our odels the poly ers are treated as thin, se i-flexible Hookean cables i ersed in a viscous liquid, and the odels differ in the description of hydrodyna ic coupling. In the first odel the coupling is based on a ultiscale dissipative particle dyna ics sche e on a Lagrangian or a hybrid Eulerian-Lagrangian Voronoi lattice, and the second odel e ploys boundary integral ethods to reduce the hydrodyna ic coupling to a potential-like contribution to the olecular forces between poly er chains. The co plexity of the for er odel is at least $\mathcal{O}(n^3)$, where n is the size of discretisation of poly er chains, whereas the latter can be reduced to $\mathcal{O}(n \log n)$ using a treecode sche e and is thus better suited to large scale si ulations. So e rheological equations of state corresponding to certain asy ptotic li its of the latter odel will also be derived.

Jan Rosenzweig, Centre for Co putational Science, Queen Mary, University of London, Mile End Road, London E1 4NS

New Coarse Spaces and Schwarz Method with Harmonic Overlaps

Marcus Sarkis

Abstract. We introduce and analyze new Schwarz preconditioners for solving elliptic proble s on unstructured grids. We construct the preconditioner by co bining in one preconditioner the additive overlapping Schwarz ethod and the iterative

substructuring ethod. We use the iterative substructuring techniques to ake functions har onic in the overlapping regions. As a result, co unication cost is s aller, PCG takes less nu ber of iterations than the corresponding Schwarz preconditioners, and allows the use of inexact solvers in the nonoverlapping regions. We also enhance the preconditioner with a new, effective, easy-to-construct coarse space. Part of this research is in joint work with Xiao-Chuan Cai and Maksy ilian Dryja. Nu erical results and theory will be presented.

Factorization of Structured Symmetric Matrices

Sanja Singer and Saša Singer

Abstract. One-sided Jacobi-like algorith can be the ethod of choice for accurate co putation of eigenvalues of an indefinite sy etric atrix A. In so e applications A is given i plicitly by its factors. These factors ay be unsuitable for the Jacobi process.

For exa ple, $A = B^T C + C^T B$ is a such atrix with i plicitly given factors, i.e.

$$A = \begin{bmatrix} B^T & C^T \end{bmatrix} \begin{bmatrix} 0 & I \\ I & 0 \end{bmatrix} \begin{bmatrix} B \\ C \end{bmatrix} := G^T \hat{J} G.$$

It is obvious that G^T is not a full colu n-rank atrix and, therefore, unsuitable for the one-sided Jacobi algorith .

To avoid explicit co putation of A, we can re-factorize A by using the indefinite QR factorization. Since the indefinite QR requires $J = \mathrm{diag}(j_{11}, j_{22}, \ldots, j_{mm})$, $j_{kk} \in \{-1, 1\}$, instead of $\hat{J}$, we first have to construct suitable unitary atrices U for the block-diagonalization of

$\hat{J}$. The initial proble is then transfor ed into

$$G^T \hat{J} G = (G^T U^T) J (UG)$$

and the indefinite QR factorization of UG gives the full colu n rank factor of A.

We describe the atrices U that can be used to block-diagonalize $\hat{J}$, and then analyze the perturbation and error bounds for the co puted UG.

Sanja Singer, Faculty of Mechanical Engineering and Naval Architecture, University of Zagreb, I. Lučića 5, 10000 Zagreb, Croatia
e– ail: ssinger@math.hr

Saša Singer, Depart ent of Mathe atics, University of Zagreb, P.O. Box 335, 10002 Zagreb, Croatia.
e– ail: singer@math.hr

Relative Perturbation Theory for Hyperbolic Singular Value Decomposition

Ivan Slapničar and Ninoslav Truhar

Abstract. We give relative perturbation bounds for singular values and perturbation bounds for singular subspaces of a hyperbolic singular value deco position for the pair (G, J), where G is a $m \times n$ full rank atrix and J is a $n \times n$ diagonal atrix of signs. The HSVD is defined as

$$G = U\Sigma V^{-1},$$

where U is unitary, Σ is diagonal with positive diagonal ele ents, and $V^*JV = P^TJP$ for so e per utation atrix P. Diagonal entries of Σ are the hyperbolic singular values, and the colu ns of U and V are the corresponding singular vectors.

The HSVD is a sound nu erical way to solve the downdating proble : the Her itian eigenvalue proble for the atrix $AA^* - BB^*$ can be solved as the HSVD for the pair (G, J), where

$$G = \begin{bmatrix} A & B \end{bmatrix}, \qquad J = \begin{bmatrix} I & \\ & -I \end{bmatrix}.$$

The HSVD is also part of a highly accurate algorith for the sy etric eigenvalue proble .

We consider two types of relative perturbations: $G + \delta G = (B + \delta B)D$ and $G + \delta G = \bar{D}(\bar{B} + \delta\bar{B})$, depending whether G has full colu n or full row rank, respectively. In both cases we also consider relative ele ent-wise perturbations of G which typically occur in nu erical co putations.

To get the flavor of our results, consider G with full colu n rank. Define

$$\beta = \|\delta B B^\dagger\|, \quad \beta_F = \|\delta B B^\dagger\|_F, \quad \psi = \frac{2\,\beta_F + \beta_F^2}{\sqrt{1 - 2\,\beta - \beta^2}},$$

where $B^\dagger$ denotes the pseudo-inverse. In particular, for the ele ent-wise perturbation of the for $|\delta G| \le \epsilon |G|$ with diagonal D such that B has unit colu ns, we have $\beta \le \epsilon\sqrt{n}/\sigma_{\min}(B)$. Assu e the unperturbed and the perturbed hyperbolic singular values, σ_i and $\tilde{\sigma}_i$, respectively, are in the sa e order. A relative Wielandt–Hoff an-type perturbation bound is

$$\sqrt{\sum_{i=1}^{n} \left(\frac{\sigma_i^2 - \tilde{\sigma}_i^2}{\sigma_i \tilde{\sigma}_i} \right)^2} \le \psi \left(\frac{1}{2}\psi + \sqrt{1 + \frac{1}{4}\psi^2} \right).$$

Thus, si ilarly to the classical SVD, the perturbation will be s all if B is well-conditioned.

Ivan Slapničar, Faculty of Electrical Engineering, Mechanical Engineering and Naval Architecture, University of Split, R. Boškovića b.b., 21000 Split, Croatia.
e– ail: ivan.slapnicar@fesb.hr
http://www.fesb.hr/∼slap

Nonlinear Problems in Dynamics by the Finite Element Method in Time Domain

Nenad Kranjčević, Milenko Stegić and Nikola Vranković

Abstract. Exact solutions of nonlinear dyna ic equations are very rare and al ost all of the ethods for solving nonlinear differential equations are only approxi ate. A general solution ethod of syste s with co plex and strong nonlinearity is the nu erical ti e integration. Nu erical integrations are usually very ti e-consu ing and ay encounter nu erical difficulties when the nonlinearity beco es very strong. A very efficient ethod for solving nonlinear differential equations in the frequency do ain is the har onic balance ethod. When the assu ption of do inance of pri ary resonance in the response is satisfied, the har onic balance ethod is very accurate and nu erically reliable ethod. If the influence of higher har onics in the response is significant, this ethod beco es very unreliable. The incre ental har onic balance ethod is ore convenient for deter ining frequency response characteristics, because a new solution can be sought, with the previous solution used as a very good approxi ation. Another approach for solving nonlinear proble s in dyna ics is the finite ele ent in ti e ethod, which is based on a weak for of Ha ilton principle of varying action [1]. Si ilar to the standard finite ele ent technique, the ti e interval is divided into a finite nu ber of ti e ele ents. The solution for all the spatial degrees of freedo at all ti e steps within a given ti e interval is sought through a set of algebraic equations. Further ore, the straightforward deter ination of the stability of solution is the second advantageous feature of this ethod. The stability of periodic solution is investigated considering s all perturbations of the solution. According to Floquet-Liapunov theore the stability can be deter ined by the eigenvalues of the transition atrix that relates the initial and final perturbations. The odel of a three-degree-of-freedo se i-definite syste with clearances under har onic excitations is used to study the feasibility of the finite ele ent in ti e ethod. The stability of solutions is analyzed using Floquet-Liapunov theore . Close agree ent is found between obtained results and the published findings of a har onic balance ethod [2].

References

[1] Y. Wang. Dynamics of unsymmetric piecewise-linear/non-linear systems using finite elements in time. *Journal of Sound and Vibration, 185, 155-170, (1995).*

[2] C. Padmanabhan and R. Singh. Spectral coupling issues in a two-degree-of-freedom system with clearance non-linearities. *Journal of Sound and Vibration, 155, 209-230, (1992).*

Milenko Stegić, Faculty of Mechanical Engineering and Naval Architecture University of Zagreb, Zagreb, Croatia
e–mail: mstegic@fsb.hr

A model of irregular curved rods

Josip Tambača

Abstract. In this work we discuss the problem of the equilibrium of elastic curved rods for rods with the piecewise smooth middle curve.

The middle curve of the irregular rod can be approximated by the family of smooth curves. We consider the asymptotics of the solution of the one–dimensional model for rods with the smooth middle curve when the middle curve tends to the irregular one. The limit function is identified as the unique solution of a certain one–dimensional boundary value problem called a model of irregular curved rods.

The obtained model is compared to the model of the junction between two straight rods, meeting at the right angle, derived directly from the equations of the three-dimensional linearized elasticity as the thickness tends to zero.

Josip Tambača, Department of Mathematics, University of Zagreb, Bijenička cesta 30, Zagreb
e–mail: tambaca@math.hr

Optimising the solution of a Ljapunov equation

Ninoslav Truhar and Krešimir Veselić

Abstract. We consider a second order damped-vibration equation $M\ddot{x} + C\dot{x} + Kx = 0$ where M, C, K (called mass, damping, stiffness matrix, respectively) are real, symmetric matrices of order n with M, K positive definite and C positive semidefinite, or equivalently $\dot{y} = \mathbf{A}y$, where $\mathbf{A}$ is obtained from M, C, K.

In this case the Ljapunov equation

$$\mathbf{A}^T X + X\mathbf{A} = -B,$$

is uniquely solvable for any B. We optimize the solution X in the following sense. Let $\mathbf{A}$ be written as $\mathbf{A} = \mathbf{A_0} + \epsilon\mathbf{D}$, where $\mathbf{A}_0$ skew-symmetric, $\mathbf{D}$ is positive semidefinite, $\epsilon > 0$. We call the parameter ϵ optimal if

$$Tr(QX) = \min,$$

where Q is a suitable chosen positive semidefinite matrix.

We obtain the for ula

$$Tr(QX) = \frac{X_1}{\epsilon} - X_0 - \epsilon\hat{X}_1 + \epsilon\sum_{i=1}^{s}\frac{\lambda_i\left(\lambda_i X_i - \epsilon Y_i\right)}{\lambda_i^2+\epsilon^2},$$

which is easy to ini ize. The constants in this for ula are obtained fro solving two auxiliary sy etric eigenvalue proble s. In particular, neither the eigenvalues nor the eigenvectors of $\mathbf{A}$ are used in this approach. This opti isation is sensibly ore efficient fro the standard ethods (Bartels–Stewart), if r is s all.

Ninoslav Truhar, Lehrgebiet Mathe atische Physik, Fernuniversität, 58084 Hagen Ger any (currently on leave fro University Josip Juraj Stross ayer, Faculty of Civil Engineering, Osijek, Croatia, e- ail: truhar@gfos.hr)
e– ail: ninoslav.truhar@fernuni-hagen.de

Krešimir Veselić, Lehrgebiet Mathe atische Physik, Fernuniversität, 58084 Hagen Ger any
e– ail: kresimir.veselic@fernuni-hagen.de

On the convergence of the Newton iteration

K. Veselić

Abstract. We give a new convergence criterion for the classical Newton ethod for finding the zero of a vector ap which is the gradient of a convex function. Our condition does not need the usual continuity of the derivative and it consists of only one bound with a constant, which is shown to be sharp. This enables us to extend the convergence result to the cases where the derivative is defined erely as a distribution.

K. Veselić, Lehrgebiet Mathe atische Physik, Fernuniversität Hagen, Postf. 940, 58084 Hagen Ger any
e– ail: kresimir.veselic@fernuni-hagen.de

On principal eigenvalue of stationary diffusion problem with nonsymmetric coefficients

Marko Vrdoljak

Abstract. We consider the eigenvalue proble

$$\begin{cases} -\mathrm{div}(\mathbf{A}\nabla \mathrm{u}) = \lambda\rho \mathrm{u} \\ u \in \mathrm{H}_0^1(\Omega) \end{cases}$$

where $\Omega \in \mathbf{R}^d$ is open and bounded, $\rho \in \mathrm{L}^\infty(\Omega)$ and $\mathbf{A} \in \mathrm{L}^\infty(\Omega; \mathrm{M}_{d\times d})$ satisfying

$$\mathbf{A}(x)\xi \cdot \xi \geq \alpha \xi \cdot \xi \,, \quad \rho(x) \geq c \,, \qquad \xi \in \mathbf{R}^d, \text{ a.e. } x \in \Omega$$

for so e $\alpha, c > 0$.

Using the strong axi u principle, obtained by Harnack's inequality, and Krein-Rut an's theore , the existence of principle eigenvalue is proved. Moreover, under appropriate conditions, the principal eigenvalue depends continuously on coefficients with respect to H-topology for $\mathbf{A}$ and L^∞ weak $*$ topology for ρ.

Marko Vrdoljak, Depart ent of Mathe atics, University of Zagreb

High-Order ENO and WENO Schemes with Flux Gradient and Source Term Balancing

Senka Vuković ans Luka Sopta

Abstract. We developed a new set of nu erical sche es particularly designed for hyperbolic conservation laws with significant source ter . These sche es are based on one hand on the essentially non-oscillatory (ENO) sche es and weighted essentially non-oscillatory (WENO) sche es (authors Harten, Osher, Engquist, Chakravarthy, Shu, Balsara) and on the other hand on the concept of the flux gradient and source ter balancing (Ber udez, Vzquez, Hubbard, LeVeque). In this paper we present results of extended nu erical testing of the original ENO and WENO sche es and the new sche es on one-di ensional shallow water equations. We perfor co putations using 2-step and 3-step Runge-Kutta ti e operator approxi ation and 3rd to 6th for al order reconstruction via pri itive function for the space operator. On the basis of the obtained nu erical results we exa ine effects of the sche e order increasing in interaction with the introduced i prove ent by the source ter deco position. We also discuss the ti e evolution of the nu erical error due to variable bed depth in quiescent flow, steady state flow and unsteady flow for the original and for the newly developed sche es.

Senka Vuković, University of Rijeka, 51000 Rijeka, Vukovarska 58, Croatia
e– ail: senka.vukovic@ri.hinet.hr

Luka Sopta, University of Rijeka, 51000 Rijeka, Vukovarska 58, Croatia
e– ail: sopta@rijeka.riteh.hr