Sveučilište u Dubrovniku
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Logika, skupovi i diskretna matematika

Exercise 3

The assignments are due to 21.11.2005.

Tutorial 3.1

1. For each of the following formulas, write "B" for "bound" or "F" for "free" under each occurence of each variable.
   (a) $\exists y(x < y)$
   (b) $(\forall x(x > 0)) \wedge (\exists y(y = x))$
2. Show $\neg\exists x\, P(x) \equiv \forall x\, \neg P(x)$ for any predicate $P(x)$.
3. Show that every statement of the form
$$((\forall x P(x))) \vee (\forall x Q(x))) \Rightarrow (\forall x(P(x) \vee Q(x)))$$
   for two predicates $P(x)$ and $Q(x)$ is true.
4. Show $(\forall x \forall y\, P(x, y)) \equiv (\forall y \forall x\, P(x, y))$ for any predicate $P(x, y)$.

Tutorial 3.2

1. Find all integers $d > 0$ such that $12 \mid d$ and $d \mid 260$.
2. Proof the following statement: If $d \mid a$ and $d \mid b$ then $d \mid (ax + by)$, for any integers x and y.
3. Find q and r guaranteed by the division algorithm such that $a = bq + r$ for $a = 28$ and $b = 160$.
4. For any integer n, prove that 2 divides $n(n + 1)$.
5. Prove that 4 does not divide $n^2 + 2$, for any integer n.
6. Prove that every square integer is of the form $4k$ or $4k + 1$, where k is an integer.
7. Proof parts (8) and (9) of Lemma 3.11.
8. Find $\gcd(44, 110)$, and $\mathrm{lcm}(6, 21)$.
9. Suppose $a > 1$ is an integer. Find $gcd(a, a + 1)$.
10. Suppose $a > 1$ is an odd integer. Find a formula for $\mathrm{lcm}(a, a + 2)$.
11. Compute $\gcd(363, 720)$ by the Euclidean algorithm.

Homework assignment 3.1 **10 points**

Solve homework assignments 2.4 and 2.5 if you have not done them already.

Homework assignment 3.2 **6 points**

1. Translate the following statements into reasonably good English/Croatian statements that involve no variables.
   (a) $\exists x \forall y (y \text{ loves } x)$ (domain of human beings)
   (b) $\forall y \exists x (y \text{ loves } x)$ (domain of human beings)
   (c) $\forall x \forall y \exists z (x + z = y)$ (domain of numbers)
   (d) $\exists x \exists y \forall z (x + z = yz)$ (domain of numbers)
   (e) $\forall x \exists y \forall z (z \geq y \Rightarrow z > x)$ (domain of numbers)
2. For each of the following formulas, write "B" for "bound" or "F" for "free" under each occurence of each variable.
   (a) $\exists z \forall y (z = y \lor y = x)$
   (b) $(\exists z (x + y = z)) \lor ((\forall x (x > 2)) \Rightarrow (\exists y (y = z)))$
3. Provide an example for two predicates $P(x)$ and $Q(x)$ such that the statement
$$(\forall x (P(x) \lor Q(x))) \Rightarrow ((\forall x\, P(x)) \lor (\forall x\, Q(x)))$$
   is false.

Homework assignment 3.3 **10 points**

1. Find all integers $d > 0$ such that $12 \mid d$ and $d \mid 240$.
2. Show that 3 divides $n(n+1)(n+2)$ for any integer n.
3. Prove that if 3 does not divide an odd integer n, then 24 divides $n^2 - 1$.
4. Find $\mathrm{lcm}(10, 15)$, $\gcd(99, 100)$, $\gcd(24, 78)$, and $\mathrm{lcm}(24, 78)$.
5. Suppose $a > 1$ is an even integer. Find a formula for $\mathrm{lcm}(a, a + 2)$.
6. Proof parts (3) and (6) of Lemma 3.11.
7. Compute $\gcd(2730, 1001)$ and $\mathrm{lcm}(2730, 1001)$ using the Euclidean algorithm.