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ON THE EQUIVALENCE OF SOME PROPERTIES WEAKER THAN COMPACTNESS

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Abstract. We give a simple proof of the equivalence of the concepts: generalized absolutely closed, almost compact and nearly C-compact. For Hausdorff spaces the above are equivalent to absolute closure.

Introduction

A Hausdorff space is called *absolutely closed* if it is closed in every Hausdorff space in which it is embedded. This concept was first introduced by Alexandroff and Urysohn [2]. The term *H-closed* was used by Alexandroff and Hopf [1]. In Hausdorff spaces absolute closure is equivalent to the condition that *every open filter base has non--void adherence* ([5], theorem 2 and [3], part I, p. 108). In not necessarily Hausdorff spaces the above condition was called *generalized absolute closure* by Chen-Tung Liu [4] and earlier on, *property* $H(i)$, by Scarborough and Stone [6], who first formulated absolute closure for non-Hausdorff spaces. In Hausdorff spaces *almost compactness* (definition (*ii*) below) is equivalent to absolute closure ([1], p. 90) and in non-Hausdorff spaces to generalized absolute closure ([6]). Sharma and Namdeo [7] introduced *nearly C-compact spaces* as a generalization to Viglino's C-compactness [9] an gave an incorrect proof of the equivalence of nearly C-compact and almost compact.

In the theorem below we give a simple proof of the equivalences mentioned above.

Definitions. In an arbitrary topological space X we define

(*i*) X is *generalized absolutely closed* if every open filter base has non-void adherence ([6], property $H(i)$ and [4], Definition 1.5).

(*ii*) X is *almost compact* if every open cover of X has a finite subfamily whose union is dense in X ([8], property (A) and [7], Definition 4).

(*iii*) X is *nearly C-compact* if for each regular closed $F \subset X$ and open cover $\{U_a\}$ of F there exists a finite subfamily of $\{U_a\}$, the closures of whose members cover F ([7], Definition 2).

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THEOREM. *In any topological space X the following are equivalent:*

(1) *X is generalized absolutely closed,*

(2) *X is almost compact,*

(3) *X is nearly C-compact.*

Proof. (1) $\Rightarrow$ (2): Suppose there exists an open cover $\mathscr{U}$ of X such that no finite subfamily has a union which is dense in X. Then $\mathscr{F} = \{X - \cup \{\overline{U} \mid U \in \mathscr{V}, \mathscr{V}$ a finite subfamily of $\mathscr{U}\}\}$ is an open filter base on X with empty adherence.

(2) $\Rightarrow$ (3): Let F be a regular closed subset of X and $\mathscr{U}$ any open cover of F. Then $\mathscr{U} \cup \{X - F\}$ is an open cover of X and hence there exists a finite subfamily $\{U_1, \ldots, U_n\}$ of $\mathscr{U}$ such that

$$X = (\bigcup_{i=1}^{n} \overline{U}_i) \cup (\overline{X - F})$$

which implies

$$\operatorname{int} F \subset \bigcup_{i=1}^{n} \overline{U}_i$$

and hence

$$F \subset \bigcup_{i=1}^{n} \overline{U}_i.$$

(3) $\Rightarrow$ (1): Let $\mathscr{F}$ be any open filter base with adherent set A, let R be any regular open set containing A. The family $(\{X - \overline{F} \mid F \in \in \mathscr{F}\}$ is an open cover of the regular closed $X - R$ so has a finite subfamily $\{X - \overline{F}_i \mid i = 1, \ldots, n\}$ satisfying

$$X - R \subset \bigcup_{i=1}^{n} \overline{X - \overline{F}_i} \subset \bigcup_{i=1}^{n} X - F_i$$

$$R \supset \bigcap_{i=1}^{n} F_i \supset F, \text{ some } F \in \mathscr{F}.$$

Thus $R \neq \emptyset$ and hence $A \neq \emptyset$.

COROLLARY. *In any Hausdorff space X the following are equivalent:*

(*a*) *X is absolutely closed,*

(*b*) *H is almost compact,*

(*c*) *X is nearly C-compact.*

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EKVIVALENTNOST NEKIH SVOJSTAVA SLABIJIH OD KOMPAKTNOSTI

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Sadržaj

U članku je dan jednostavan dokaz da su ekvivalentni ovi pojmovi: generalizirano apsolutno zatvoren, skoro kompaktan i gotovo *C*-kompaktan. Za Hausdorffove prostore ti su pojmovi ekvivalentni apsolutnoj zatvorenosti.