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CLOSED GRAPHS ON CONVERGENCE SPACES

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Abstract. This is the fourth in a series of investigations answering various unsolved problems relative to convergence spaces. The major goal of this present research is to develop the convergence space theory of maps with closed graphs, answer some previously open questions and to show that many recent results relative to generalizations of the topological concept of the closed graph such as the strongly closed graph or maps with property S are in reality simple corollaries to the appropriate convergence space proposition. This yields more evidence that the proper structure in which to study the basic properties of such mappings is not the topological space, but is the (pre)convergence space.

1. Introduction

Certain results which appear in this paper have been briefly reported upon in [9]. This article contains the complete proofs for the results briefly summarized in [9] as well as definitions, examples and additional applications and results which are of some interest to the general topologist.

In 1974, I introduced in [3] a nonstandard generalization for all of the perfect type maps in the sense of Whyburn [17] and Iliadis [10] which have ever appeared in the mathematical literature. At a later date, a slightly better generalization was devised for perfect type maps relative to standard (pre)convergence spaces in the sense of Kent [13]. Following the appearance of the results in [7], [8] many questions have been raised as to the relations, if any, between these generalized perfect maps and such concepts as the closed or compact maps and the theory of maps with a closed graph relative to the fundamentally important preconvergence structures. The basic reason for the apparent interest in these new concepts lies in the fact that so »many« perfect type maps exist. For example, if (X, q), (Y, p) are preconvergence spaces and $f : X \to Y$ is a weakly-continuous injection, then $f^{-1} : (f[X], p') \to (X, q)$ is a perfect map. Moreover, it is becoming more and more obvious that the proper structure in which to study the basic properties of such mappings is not the topological space, but is the preconvergence space [7], [8]. The basis for this last conclusion lies in the fact, as exemplified above, that many, but not

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all results relative to perfect [10], ϑ-perfect [1], δ-perfect [15] maps and the like which have recently appeared in the literature are simple corollaries to the more general preconvergence space propositions as they appear in [3], [4], [7], [8], among other places.

This paper is the first in a series dealing with the relation between perfect, compact, closed maps; maps with closed graphs and the like concepts for preconvergence spaces. We also answer some questions left open in [8]. In this paper we characterize maps with closed graphs and briefly indicate how these results may be applied to numerous topological areas, such as the theory of strongly closed graphs. In particular, a major result shows that for preconvergence spaces $(X, q), (Y, p)$, a map $f : X \to Y$ has a closed graph if and only if whenever a filter $\mathscr{F}$ q-converges to $x \in X$ and the filter $f(\mathscr{F})$ p-converges to $y \in Y$, then $f(x) = y$.

2. Preliminaries

We adhere to the following notational conventions and basic definitions. For a set X, $F(X)$ (resp. $U(X)$) denotes the set of all filters (resp. ultrafilters) on X. If nonempty $A \subset \mathscr{P}(X)$, the power set of X, and A has the finite intersection property, then $[A]$ denotes the filter generated by A. If $\emptyset \neq A \in \mathscr{P}(X)$, then $[A]$ is the *principal filter* generated by A. In [13] Kent defines a *convergence function* on X to be a map $q : F(X) \to \mathscr{P}(X)$ such that

(CS1) for each $x \in X$, $x \in q([\{x\}])$,

(CS2) if $\mathscr{F}, \mathscr{G} \in F(X)$ and $\mathscr{F} \subset \mathscr{G}$, then $q(\mathscr{F}) \subset q(\mathscr{G})$.

Throughout this paper the pairs (X, q), (Y, p), where q and p are convergence functions, are called *preconvergence spaces.* A filter $\mathscr{F} \in F(X)$ is said to *q-converge to $x \in X$* if $x \in q(\mathscr{F})$ and we sometimes denote this by $\mathscr{F} \underset{q}{\to} x$ or simply by $\mathscr{F} \to x$.

For a map $f : (X, q) \to (Y, p)$ and $\mathscr{F} \in F(X)$, let $f(\mathscr{F}) = [\{f[F] \mid F \in \mathscr{F}\}]$. Moreover, if $\mathscr{F} \in F(X)$ and for each $F \in \mathscr{F}$, $F \cap f[X] \neq \emptyset$, then $f^{-1}(\mathscr{F}) = \{f[F] \mid F \in \mathscr{F}\} \in F(X)$. Also $G(f) = \{(x, f(x)) \mid [x \in X] \wedge [f(x) \in Y]$ denotes the *graph of f*.

A non-Kuratowski type closure operator is also used for preconvergence space investigations. Let $A \subset X$, then $\mathrm{cl}_q(A) = \{x \mid (x \in X) \wedge \exists y ((y \in U(X)) \wedge (A \in y) \wedge (y \underset{q}{\to} x))\}$. In general $A \subset \mathrm{cl}_q(A)$ and $\mathrm{cl}_q(\mathrm{cl}_q(A)) \neq \mathrm{cl}_q(A)$. A set $A \subset X$ is *q-closed* or simply *closed* if $A = \mathrm{cl}_q(A)$.

Finally, for two filters $\mathscr{G}, \mathscr{F} \in F(X)$ such that $\mathscr{F} \cup \mathscr{G}$ has the finite intersection property, then $\mathscr{F} \vee \mathscr{G} \in F(X)$ denotes the smallest (with respect to $\subset$) filter containing $\mathscr{F}$ and $\mathscr{G}$. All other pertinent notation and definitions appear in the sequel or in the references.

3. Main results

THEOREM 3.1. *Let* $f : (X, q) \to (Y, p)$.

(*i*) *If* $G(f)$ *is closed in the product preconvergence space, then whenever* $\mathscr{F} \in F(X)$, $\mathscr{F} \to x \in X$ *and* $f(\mathscr{F}) \to y \in Y$, *it follows that* $f(x) = y$.

(*ii*) *If whenever* $\mathscr{U} \in U(X)$, $\mathscr{U} \to x \in X$ *and* $f(x) \to y \in Y$ *imply that* $f(x) = y$, *then* $G(f)$ *is closed.*

Proof. (*i*) Assume that $\mathscr{F} \in F(X)$, $\mathscr{F} \to x \in X$, $f(\mathscr{F}) \to y \in Y$ and $F \in \mathscr{F}$. Let $\pi = q \times p$ denote the usual product preconvergence space convergence function on $X \times Y$. We establish that $(x, y) \in \mathrm{cl}_\pi(F \times f[F])$. Suppose that $\mathscr{U} \in U(\mathscr{F})$, the set of all ultrafilters containing $\mathscr{F}$. Then $f(\mathscr{U}) \in U(f(\mathscr{F}))$, $\mathscr{U} \to x$ and $f(\mathscr{U}) \to y$. Since $F \in \mathscr{U}$, $f[F] \in f(\mathscr{U})$, then $F \times f[F] \in \mathscr{V}$, where $\mathscr{V} \in U(X \times Y)$ and $\mathscr{V} \to (x, y)$. Thus $(x, y) \in \mathrm{cl}_\pi(F \times f[F])$. Now $F \times f[F] \subset G(f)$ implies that $G(f) \in \mathscr{V}$. Hence $(x, y) \in \mathrm{cl}_\pi(G(f)) = G(f)$ implies that $f(x) = y$.

(*ii*) Assume that if $\mathscr{U} \in U(X)$, $\mathscr{U} \to x$ and $f(\mathscr{U}) \to y$, then $f(x) = y$. Let $(w, z) \in X \times Y - G(f)$. Then from above it follows that there does not exist some $\mathscr{V} \in U(X)$ such that $\mathscr{V} \to w$ and $f(\mathscr{V}) \to z$. Assume that $(w, z) \in \mathrm{cl}_\pi((G(f))$. Then there exists some $\mathscr{W} \in U(X \times Y)$ such that $G(f) \in \mathscr{W}$ and $\mathscr{W} \to (w, z) = (w, f(w))$. Now the first projection $P_1(\mathscr{W}) = \mathscr{U} \in U(X)$ and $\mathscr{U} \to w$. Letting $(I, f) : X \to X \times Y$ be the map defined by $(I, f)(x) = (x, f(x))$, then it follows that for each $A \subset X \times Y$, $(I, f)^{-1}[A] \subset P_1[A] \cap f^{-1}[P_2[A]]$. Thus $P_1(\mathscr{W}) \vee f^{-1}(P_2(\mathscr{W})) \subset (I, f)^{-1}(\mathscr{W})$. Hence $f^{-1}(P_2(\mathscr{W})) \subset P_1(\mathscr{W})$ implies that $P_2(\mathscr{W}) \subset f(P_1(\mathscr{W}))$. Since $P_2(\mathscr{W})$ is an ultrafilter, then $P_2(\mathscr{W}) = f(P_1(\mathscr{W})) = f(\mathscr{U}) \to f(w)$. This contradiction implies that $(w, z) \notin \mathrm{cl}_\pi(G(f))$. Since $G(f) \subset \mathrm{cl}_\pi(G(f))$ and $X - G(f) \subset X - \mathrm{cl}_\pi(G(f))$, then $\mathrm{cl}_\pi(G(f)) = G(f)$ and this completes the proof.

COROLLARY 3.1.1. *A map* $f : (X, q) \to (Y, p)$ *has a closed graph if and only if whenever* $\mathscr{F} \in F(X)$, $\mathscr{F} \to x \in X$ *and* $f(\mathscr{F}) \to y \in Y$, *then* $f(x) = y$.

Recall that a map $f : (X, q) \to (Y, p)$ is *weakly-continuous* (resp. *continuous*) if for each $\mathscr{F} \in U(X)$ (resp. $\mathscr{F} \in F(X)$) such that $\mathscr{F} \to x \in X$, then $f(\mathscr{F}) \to f(x)$. If Y is pseudotopological and $f : X \to Y$ is weakly-continuous, then f is continuous. As usual a space (X, q) is *compact* if each $\mathscr{U} \in U(X)$ is q-convergent.

THEOREM 3.2. *If* $f : (X, q) \to (Y, p)$ *has a closed graph and* Y *is compact, then* f *is weakly-continuous.*

Proof. Let $G(f)$ be closed and Y compact. Assume that $\mathscr{U} \in U(X)$ and that $\mathscr{U} \to x \in X$. Then there exists some $y \in Y$ such that $f(\mathscr{U}) \to y$. Theorem 3.1. implies that $y = f(x)$ and the result follows.

A space is *Hausdorff* if each ultrafilter converges to at most one point. Corollary 3.2.3 in [8] shows that if (Y, p) is Hausdorff and $f : (X, q) \to (Y, p)$ is weakly-continuous, then $G(f)$ is closed in $(X \times \times Y, \pi)$. This yields the following result.

THEOREM 3.3. *Let Y be compact and Hausdorff. Then* $f : (X, q) \to \to (Y, p)$ *has a closed graph if and only if f is weakly-continuous.*

A map $f : (X, q) \to (Y, p)$ is *perfect* [8] if whenever $\mathcal{U} \in U(Y)$ and $\mathcal{U} \to y \in Y$, then for each $\mathcal{V} \in U(X)$ such that $f(\mathcal{V}) = \mathcal{U}$ there exists some $x \in f^{-1}(y)$ such that $\mathcal{V} \to x$.

THEOREM 3.4. *Let* $f : (X, q) \to (Y, p)$, *X and Y be compact, Y Hausdorff, and* $f[X]$ *closed in Y. If* $G(f)$ *is closed in* $X \times Y$, *then f is perfect.*

Proof. Theorem 3.2 implies that f is weakly-continuous. Application of Corollary 2.12.1 in [8] implies that f is perfect and the proof is complete.

For preconvergence spaces the existence of functions with a closed graph is somewhat more critical than for topological spaces. Let q be a convergence function for the set X. The *pretopological modification*, $\hat{q}$, is defined as follows: $x \in \hat{q}(\mathcal{F})$ for $\mathcal{F} \in F(X)$ if and only if $\mathcal{N}_q(x) = = \wedge \{\mathcal{U} \mid (\mathcal{U} \in U(X)) \wedge (\mathcal{U} \to x)\} \subset \mathcal{F}$.

THEOREM 3.5. *Let X and Y be compact and Y be Hausdorff. If the pretopological modification of q is a topology on X and* $\hat{p}$ *is not a topology on Y, then there does not exist a surjection* $f : (X, q) \to (Y, p)$ *with a closed graph.*

Proof. Simply assume that there exists a surjection $f : X \to Y$ such that $G(f)$ is closed. It follows that f is perfect and weakly-continuous. However, Theorem 2.8 part (*iv*) [8] states that $\hat{p}$ must be a topology on Y. The result follows from this contradiction.

Let X be an infinite set and $\mathcal{U} \in U(X)$ a free (i. e. nonprincipal) ultrafilter on X. Consider any point $z \in X$ and let $\mathcal{G}_z = \{U \cup \{Z\} \mid U \in \in \mathcal{U}\} \cup \{\{x\} \mid x \in X - \{z\}\}$. Then $\mathcal{G}_z$ is a base for a topology T_z on X.

THEOREM 3.6. *Let X be an infinite set and* $\mathcal{U} \in U(X)$ *a free ultrafilter on X. For each* $z \in X$, *the space* (X, T_z) *is a Hausdorff completely normal, fully normal, door topological space.*

Proof. Let $z, x, y \in X$, $x \neq y$, $x \neq z, y \neq z$. Then $\{x\} \cap \{y\} = = \emptyset$ and $\{x\}, \{y\} \in T_z$. Now assume that $y = z$. Since $\cap \{U \mid U \in \mathcal{U}\} = = \emptyset$, then for each $w \in X - \{z\}$ there exists some $U_w \in \mathcal{U}$ such that $w \notin U_w$. This implies that $\{w\} \cap (U_w \cup \{z\}) = \emptyset$. Consequently, T_z is Hausdorff.

Assume that $A, B \subset X$ and that $(\mathrm{cl}_X(A)) \cap B = A \cap (\mathrm{cl}_X(B)) = = \emptyset$. If $z \notin \mathrm{cl}_X(A)$, then $\mathrm{cl}_X(A) \in T_z$. Thus the T_z-open sets $\mathrm{cl}_X(A)$

and $X - \mathrm{cl}_X(A)$ are separating sets for A and B. If $z \notin B$, then $B \in \in T_z$ and the T_z-open sets B and $X - \mathrm{cl}_X(B)$ are separating sets for A and B. Thus (X, T_z) is completely normal.

Let $\mathscr{C}$ be an open cover of X. Then there exists some $G \in \mathscr{C}$ such that $z \in G$. Hence there exists some $U \in \mathscr{U}$ such that $U \cup \{z\} \subset G$. This implies that

$$\{\{x\} \mid z \neq x \in X\} \cup \{U \cup \{z\}\}$$

is an open cover of X which star refines $\mathscr{C}$. Thus X is fully normal.

Finally to show that (X, T_z) is a door space we need to show that every subset of X is either open or closed. Hence let $A \subset X$ and assume that $z \notin A$. Then $A \in T_z$. On the other hand, if $z \in A$, then $\mathrm{cl}_X(A) = A$. Therefore, (X, T_z) is a door space and this completes the proof.

Recall that a preconvergence space is T_1 if every principal ultrafilter converges to only one point. The following result is similar to but much more general then the results which appear in [11] and [12] where only topological spaces are investigated.

THEOREM 3.7. *Let (X, q) be T_1. Assume that there exists some $z \in X$ such that for every free ultrafilter $\mathscr{U} \in U(X)$ the fact that the identity map $I : (X, T_z) \to (X, q)$ has a closed graph implies that I is weakly-continuous. Then (X, q) is compact.*

Proof. Assume that X is not compact. Then there exists some non-q-convergent $\mathscr{U} \in U(X)$. Since every principal ultrafilter q--converges, then $\mathscr{U}$ is free. Let z be the hypothesized element of X. Clearly $\mathscr{U}$ does not converge to any $x \in X - \{z\}$. Consider the topological space (X, T_z) determined by $\mathscr{U}$ and z. Let $\mathscr{F} \in F(X)$ such that $\mathscr{F}$ T_z-converges to some $x \in X - \{z\}$ and, for the identity map I, $I(\mathscr{F})$ is q-convergent to y. Then $\mathscr{F}$ is a principal ultrafilter generated by x. Thus $\mathscr{F} = [\{x\}] = \{A \mid [A \subset X] \wedge [x \in A]\}$. Hence $I(\mathscr{F}) = I([\{x\}]) = [\{x\}]$ and $I(\mathscr{F})$ is a q-convergent to $x = y$ by the T_1 property. Thus $I(x) = y$. Now let $x = z$ and $\mathscr{F}$ be T_z-convergent to x. Since that T_z-neighborhood filter at z is $\{U \cup \{z\} \mid U \in \in \mathscr{U}\} = \mathscr{N}(z)$ it follows that $\mathscr{N}(z) \subset \mathscr{F}$. Assume that $I(\mathscr{F}) = \mathscr{F}$ is q-convergent to $y \in X$ and let $\mathscr{V} \in U(\mathscr{F})$. Then $\mathscr{V}$ is q-convergent to y, and $\mathscr{N}(z) \subset \mathscr{V}$ (i. e. $y \in \mathrm{ad}_q(\mathscr{N}(z))$). From the construction of $\mathscr{N}(z)$, it follows that either $\mathscr{V} = \mathscr{U}$ or $\mathscr{V} = [z]$. However, since $\mathscr{U}$ does not q-converge, then this implies that $\mathscr{V} = [z]$. Therefore from the T_1 property it follows that $z = y$. Consequently, $I(z) = y$ and Theorem 3.1 imply that I has a closed graph. From the hypothesis, it follows that I is weakly-continuous. Since $\mathscr{U}$ is T_z-convergent to z then $I(\mathscr{U}) = \mathscr{U}$ is q-convergent of $I(z) = z$. This contradiction completes the proof.

COROLLARY 3.7.1. *Let $\mathcal{S}$ be class of all Hausdorff, completely normal, fully normal, door topological spaces and let (Y, p) be a T_1 preconvergence space. If for every $X \in \mathcal{S}$ every bijection $f : X \to (Y, p)$ with a closed graph is weakly-continuous, then Y is compact.*

Remark. Obviously, Theorem 3.2 is a very strong converse to Theorem 3.7.

4. Applications

In this section we are interested in applying the above results to concepts which have recently become of interest to the general topologist. In order to accomplish this we need only show that pertinent topological concepts are characterized by specific preconvergence spaces. Once this has been accomplished, then the reader can simply translate our previous results into the appropriate topological language.

Recall that if $f : (X, q) \to (Y, p)$ is weakly-continuous and p is a pseudotopological convergence structure, then f is continuous. Let τ, T be topologies defined on X and Y respectively. A filter $\mathcal{F} \in F(X)$ will δ-converge to $x \in X$ in the sense of Veličko [16] if for each regular-open $G \in \tau$ containing x there exists some $F \in \mathcal{F}$ such that $F \subset G$. The *δ-convergence structure* is a topological convergence structure determined by the semiregularization τ_s of τ. The topology τ_s is the topology generated by the set of all regular-open subsets of X. A filter $\mathcal{F}$ is *ϑ-convergent* to $x \in X$ [8] if for each open neighborhood G of x there exists some $F \in \mathcal{F}$ such that $F \subset \mathrm{cl}_X(G)$, where the closure is with respect to τ. Let $C(X)$ be the set of all real valued continuous functions defined on (X, τ) and τ_w the weak topology on X generated by $C(X)$. Then a filter $\mathcal{F} \in F(X)$ is *w-convergent* to $x \in X$ if $\mathcal{F}$ converges to x in the τ_w topology [5]. Finally, let $\mathscr{C}_r(x) = \{\mathrm{cl}_X(G) \mid (x \in \mathrm{cl}_X(G)) \wedge (G \in \tau)\}$ where $x \in X$. Then a filter $\mathcal{F} \in F(X)$ is *rc-convergent* to $x \in X$ [6] if for each $H \in \mathscr{C}_r(x)$ there exists some $F \in \mathcal{F}$ such that $F \subset H$. Observe that in the above definitions for these special types of convergence structures we could have restricted our attention to filter bases.

As previously observed the δ and w convergence structures are topological. However, the ϑ convergence structure is not in general topological but is only pretopological in character.

Numerous unusual separation properties can be characterized by these special convergence spaces. For example, (X, τ) is Hausdorff (resp. Uryshon, completely-Hausdorff) if and only if (X, δ) (resp. (X, ϑ), (X, w)) is a Hausdorff convergence space. A space (X, τ) is weakly-Hausdorff (resp. weakly-completely-Hausdorff, Hausdorff) if and only if (X, δ) (resp. (X, w), (X, ϑ)) is a T_1 convergence space. Also some unusual compactness type properties may be characterized by the compactness of convergence spaces. A space (X, τ) is nearly-compact (resp. quasi-H-closed, completely-Hausdorff, S-closed) if and only if (X, δ) (resp. (X, ϑ), (X, w), (X, rc)) is a compact convergence space. Moreover, a map $f:(X, \tau) \to (Y, T)$ is almost--continuous (resp. weakly-ϑ-continuous, ϑ-continuous, a c-map) if and only if $f:(X, \tau) \to (Y, \delta)$ (resp. $f:(X, \tau) \to (Y, \vartheta)$, $f:(X, \vartheta) \to (Y, \vartheta)$, $f:(X, \tau) \to (Y, w)$) is convergence space continuous. Other interesting topological concepts follow from the equivalence of these convergence structures. For example, (X, τ) is regular (resp. almost--regular, semiregular, extremely disconnected) if and only if τ-convergence is equivalent to ϑ-convergence (resp. δ-convergence is equivalent to ϑ-convergence, τ-convergence is equivalent to δ-convergence, rc-convergence is equivalent to ϑ-convergence). Relative to the previous results established in this paper, we have that a map $f:(X, \tau) \to (Y, T)$ has a strongly closed graph (resp. property S) if and only if $f:(X, \tau) \to (Y, \vartheta)$ (resp. $f:(X, \tau) \to (Y, \delta)$) has a convergence space closed graph. Also, where we do not assume continuity or surjectivity, it follows that $f:(X, \tau) \to (Y, T)$ is ϑ-perfect [1] (resp. δ-perfect [15]) if and only if $f:(X, \vartheta) \to (Y, \vartheta)$ (resp. $f:(X, \delta) \to (Y, \delta)$) is convergence space perfect.

The reader may now translate each of the results in section three into the corresponding topological language associated with these special types of convergence spaces. For example, let (X, τ) be regular compact and (Y, T) Hausdorff nearly-compact. If a surjection $f: X \to Y$ has a strongly closed graph, then f is ϑ-perfect.

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ZATVORENI GRAFOVI NA PROSTORIMA KONVERGENCIJE

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Sadržaj

Ovo je četvrti članak u seriji istraživanja da se odgovori na razna neriješena pitanja vezana za prostore konvergencije. Glavni cilj ovog rada je razvijanje teorije preslikavanja sa zatvorenim grafovima što daje odgovore na neka otvorena pitanja i ujedno generalizira neke novije rezultate koji se odnose na poopćenja topološkog pojma zatvorenog grafa.