Seminar on Number Theory and Algebra
&
Department of Mathematics, Faculty of Science,
University of Zagreb

Book of Abstracts

Workshop on Number Theory and Algebra

on the occasion of 60^{th} birthday of Professor Ivica Gusić

November 26 – 28, 2014, Zagreb, Croatia

Preface

This brochure consists of the program of presentations and abstracts of papers presented at the *Workshop on Number Theory and Algebra* organized by *Seminar on Number Theory and Algebra* and *Department of Mathematics, Faculty of Science, University of Zagreb.* The workshop is organized on the occasion of 60^{th} birthday of Ivica Gusić, a Croatian mathematician with important scientific contributions to number theory and algebra, in particular to arithmetic of elliptic curves and problems on decomposability of polynomials. It is held at Department of Mathematics, University of Zagreb, Bijenička cesta 30, from November 26 to November 28, 2014.

The scientific program of this workshop consists of invited talks of international and Croatian experts in number theory and algebra, dedicated to the areas of the research interests of Ivica Gusić and other members of Croatian number theory and algebra groups.

Index of all authors is given at the end of the brochure.

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Conference Language: English

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Dragutin Svrtan, University of Zagreb
Marko Tadić, University of Zagreb
Petra Tadić, University of Pula
Robert Tichy, Graz University of Technology

Program

The conference is held at Department of Mathematics, University of Zagreb, Bijenička cesta 30. All talks take place in room A101 on the first floor.

Wednesday, November 26, 2014

10:00 - 10:15	Opening and Announcements
Morning session (chairman: Andrej Dujella)	
10:15 - 10:55	*Common expansions in noninteger bases* Attila Pethő
11:00 - 11:40	*Some connections between non-commutative harmonic analysis and the theory of automorphic forms* Marko Tadić
11:40 - 12:05	coffee break
12:05 - 12:45	*On lacunary polynomials and their number of terms* Clemens Fuchs
12:45 - 14:45	lunch break
Afternoon session 1 (chairman: Ákos Pintér)	
14:45 - 15:10	*On polynomial values of sums of Fibonacci numbers* Thomas Stoll
15:15 - 15:40	*Whittaker modules for the affine Lie algebra* $A_1^{(1)}$ Dražen Adamović
15:45 - 16:10	*The difference graphs of* S*-units* Dijana Kreso
16:10 - 16:35	coffee break
Afternoon session 2 (chairman: Lajos Hajdu)	
16:35 - 17:00	*Derivations, homomorphisms and elementary operators* Dijana Ilišević
17:05 - 17:30	*Householder's approximants and continued fraction expansion of quadratic irrationals* Vinko Petričević
17:35 - 18:00	*High rank elliptic curves with prescribed torsion group over quadratic fields* Mirela Jukić Bokun

Thursday, November 27, 2014

Morning session (chairmain: Attila Pethő)	
10:00 - 10:40	*Polynomial Diophantine equations* Robert Tichy
10:45 - 11:25	*Elliptic curves and Diophantine triples* Juan Carlos Peral
11:25 - 11:50	coffee-break
11:50 - 12:30	*Finiteness results for F-Diophantine sets* Lajos Hajdu
12:30	conference photo
12:45 - 14:45	lunch break
Afternoon session 1 (chairman: Robert Tichy)	
14:45 - 15:10	*Some recent results on additive decomposition of sets* Christian Elsholtz
15:15 - 15:40	*Rogers-Ramanujan type identities and quasiparticles in the principal picture of $\widehat{\mathfrak{sl}}_2$* Mirko Primc
15:45 - 16:10	*Injectivity of the specialization homomorphism of elliptic curves* Petra Tadić
16:10 - 16:35	coffee break
Afternoon session 2 (chairman: Florian Luca)	
16:35 - 17:00	*On the existence of Diophantine quintuples* Alan Filipin
17:05 - 17:30	*$D(-1)$-triples of the form $\{1,b,c\}$ in the ring $\mathbb{Z}[\sqrt{-t}], t>0$* Ivan Soldo
17:35 - 18:00	*Two divisors of $(n^2+1)/2$ summing up to $\delta n+\varepsilon$, for δ and ε even* Sanda Bujačić
19:00	conference dinner

Friday, November 28, 2014

Morning session (chairmain: Juan Carlos Peral)	
10:00 - 10:40	*Around the Littlewood conjecture* Yann Bugeaud
10:45 - 11:25	*On the decomposability of the linear combinations of Bernoulli polynomials* Ákos Pintér
11:25 - 11:50	coffee-break
11:50 - 12:30	*On the counting function of irregular primes* Florian Luca
12:30 - 14:30	lunch break
Afternoon session 1 (chairman: Yann Bugeaud)	
14:30 - 15:10	*Arithmetic and geometric progressions in the solution set of Diophantine equations* Attila Bérczes
15:15 - 15:40	*Modular forms, de Rham cohomology and congruences* Matija Kazalicki
15:45 - 16:10	*On generalized problem of Diophantus in some number fields* Zrinka Franušić
16:10 - 16:35	coffee break
Afternoon session 2 (chairman: Clemens Fuchs)	
16:35 - 17:00	$D(4)$*-pair* $\{a,b\}$ *and its extension* Ljubica Baćić
17:05 - 17:30	*On the Mordell-Weil group of elliptic curves induced by families of Diophantine triples* Miljen Mikić
17:35 - 18:00	*Torsion of rational elliptic curves over number fields* Enrique González Jiménez

Abstracts of talks

Whittaker modules for the affine Lie algebra $A_1^{(1)}$

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Abstract. We will present a complete description of Whittaker modules for the affine Lie algebras $A_1^{(1)}$ at arbitrary level. A particular emphasis will be put on explicit realization of Whittaker modules at the critical level.

This is a joint work with R. Lu and K. Zhao.

$D(4)$-pair $\{a, b\}$ and its extension

Ljubica Baćić
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Abstract. We consider the extensibility of a general $D(4)$-pair $\{a, b\}$ and prove some results that support the conjecture that there does not exist a $D(4)$-quintuple. In the proof we use the standard methods used in solving similar problems. So we firstly transform our problem into solving the system of simultaneous Pellian equations which furthermore leads to finding intersection of binary recurrence sequences. It is eventually solved combining congruence method with hypergeometric method and Baker's theory of linear forms in logarithms. Here we made Baker-Davenport reduction at the beginning and also improved Rickert's theorem in our special case.

This is a joint work with A. Filipin.

Arithmetic and geometric progressions in the solution set of Diophantine equations

Attila Bérczes
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Abstract. In 2004 Bérczes and Pethő started to investigate the arithmetic progressions in the solution set of norm form equations. Since then the investigation of special progressions appearing in the solution set of Diophantine equations has resulted in a series of interesting results.

Bérczes and Pethő, Bérczes, Pethő and Ziegler and later Bazsó determined all arithmetic progressions forming solutions of some parametric families of norm form equations. Arithmetic progressions in the solution set of Pell equations were investigated by Pethő and Ziegler, and by Dujella, Pethő and Tadić.

In this talk a survey on these results will be presented, along with some recent results obtained by Bérczes and Ziegler on geometric progressions in the solution set of Pell-equations. Further, Bérczes and Ziegler also determined all geometric progressions consisting of elements of a given Lucas sequence.

Around the Littlewood conjecture

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Abstract. The Littlewood conjecture in Diophantine approximation claims that every pair (α, β) of real numbers satisfies

$$\inf_{q \geq 1} q \cdot \|q\alpha\| \cdot \|q\beta\| = 0,$$

where $\| \cdot \|$ denotes the distance to the nearest integer. In 2004, de Mathan and Teulié asked the following analogous question: for a given prime number p, is it true that

$$\inf_{q \geq 1} q \cdot \|q\alpha\| \cdot |q|_p = 0$$

holds for every real number α? Here, $|\cdot|_p$ denotes the p-adic absolute value normalized such that $|p|_p = p^{-1}$. We present recent results towards the resolution of these two problems, which are still not solved.

Two divisors of $(n^2+1)/2$ summing up to $\delta n+\varepsilon$, for δ and ε even

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Abstract. We deal with the problem of the existence of two divisors of $(n^2+1)/2$ whose sum is equal to $\delta n+\varepsilon$, in the case when δ and ε are even, or more precisely in the case in which $\delta \equiv \varepsilon+2 \equiv 0$ or $2 \pmod 4$. We completely solve the cases $\delta=2, \delta=4$ and $\varepsilon=0$. In our proofs we use Diophantine equations and their properties, with a special accent on Pellian equations and their properties.

Some recent results on additive decomposition of sets

Christian Elsholtz
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Abstract. The question, which sets of integers can be written as a sumset $S=A+B$, possibly with some exceptions, is for most given sets S wide open. Ostmann asked it for the set of primes, Sárközy for the set of smooth numbers, and also for quadratic residues modulo p. We will give a survey of some recent results in this area.

On the existence of Diophantine quintuples

Alan Filipin
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Abstract. We call the set of m positive distinct integers a Diophantine m-tuple if the product of any of its two elements increased by 1 is a perfect

square. The folklore conjecture states that there exists no Diophantine quintuple. In this talk we will present the most recent results on the extensibility of Diophantine pairs and triples which all supports the given conjecture.

The results are joint work with M. Cipu, Y. Fujita and A. Togbé.

On generalized problem of Diophantus in some number fields

ZRINKA FRANUŠIĆ
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Abstract. The set of nonzero and distinct elements $\{z_1, z_2, z_3, z_4\}$ in R such that $z_iz_j + w$ is a perfect square in R for $1 \leq i < j \leq 4$ is called a *Diophantine quadruple with the property* $D(w)$ in R, where R denotes a commutative ring with the unity. We consider the problem of existence of Diophantine quadruples with the property $D(w)$ in rings of integers of some number fields.

On lacunary polynomials and their number of terms

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Abstract. We will discuss work in progress with Umberto Zannier and Vincenzo Mantova which shows that every solution $g(x) \in \mathbb{C}(x)$ of an algebraic equation $F(x, g(x)) = 0$, where $F(x, y) \in \mathbb{C}[x, y]$, can be represented as a quotient of two polynomials with number of terms depending only on the degree of F in y and the number of distinct terms in x. This generalizes results on 'lacunary' polynomials, which were recently obtained in a series of papers by Zannier and which originate from questions posed independently by Rényi and Erdős and a conjecture by Schinzel. In the talk I will roughly sketch the proof and discuss applications to the problem of integral points on finite covers W of $\mathbb{G}_m^n$ and to Bertini-type results for covers of tori; these and other results exploit the point of view that 'lacunary' polynomials can be viewed as restrictions of regular functions on tori to a 1-parameter subgroup or coset.

Torsion of rational elliptic curves over number fields

Enrique González Jiménez
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Abstract. Let E be an elliptic curve defined over the rationals. We study the relationship between the torsion subgroup over the rationals and the torsion subgroup over a number field of fixed degree d.

In this talk I will present joint works with J. M. Tornero ($d = 2$) and F. Najman and J. M. Tornero ($d = 3$).

Finiteness results for F-Diophantine sets

Lajos Hajdu
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Abstract. Let $F \in \mathbb{Z}[x, y]$ be a polynomial with integer coefficients and m be an integer with $m \geq 2$. A set A of positive integers is called an (F, m)-Diophantine set if $F(a, b)$ is an m-th power for any $a, b \in A$ with $a \neq b$. Further, we call a set A of positive integers an $(F, *)$-Diophantine if $F(a, b)$ is a perfect power for any $a, b \in A$ with $a \neq b$. Note that in the latter case the exponents of the powers are allowed to be different.

In the talk we shall be interested in finiteness results concerning (F, m)-Diophantine sets and $(F, *)$-Diophantine sets. First we give a brief overview of the known results (due to Bérczes, Bugeaud, Dujella, Gyarmati, Luca, Rivat, Sárközy, Stewart and many others) concerning the cases $F(x, y) = xy + 1, xy + n, x^2 + y^2, x + y$. Then we present new results for general $F(x, y)$. Among others, we give a complete characterization of primitive polynomials $F(x, y)$ for which A can be infinite.

This is a joint work with A. Bérczes, A. Dujella and Sz. Tengely.

Derivations, homomorphisms and elementary operators

DIJANA ILIŠEVIĆ
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Abstract. Among the most important classes of additive maps on rings are (generalized) derivations, homomorphisms (in particular, automorphisms) and elementary operators. The aim of this talk is to describe the intersection of any two of these three classes.

This talk is based on joint work with Daniel Eremita and Ilja Gogić.

High rank elliptic curves with prescribed torsion group over quadratic fields

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Abstract. There are 26 possibilities for the torsion group of an elliptic curve defined over quadratic number fields. In this talk we will present examples of high rank elliptic curves in most of the cases and searching methods that we used.

These results come from a joint work with A. Dujella, J. Aguirre and J. C. Peral.

Modular forms, de Rham cohomology and congruences

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Abstract. After reviewing Atkin and Swinnerton-Dyer's (ASD) speculation about existance of a "p-adic Hecke eigenbasis" for the spaces of cusp forms for non-congruence subgroups, we briefly explain the connection of this

phenomena with certain de Rham cohomology groups associated to modular forms.

In our main result, we extend ASD congruences to weakly modular forms (modular forms that are permitted to have poles at cusps). Unlike the case of original congruences for cusp forms, these congruences are nontrivial even for congruence subgroups.

As an example, we consider the space of cusp forms of weight 3 on a certain genus zero quotient of Fermat curve $X^N + Y^N = Z^N$. We show that the Galois representation associated to this space is given by a Grössencharacter of the cyclotomic field $\mathbb{Q}(\zeta_N)$. Moreover, for $N = 5$ the space does not admit a "p-adic Hecke eigenbasis" for (non-ordinary) primes $p \equiv 2, 3 \pmod 5$, which provides a counterexample to Atkin and Swinnerton-Dyer's original speculation.

This is a joint work with Anthony J. Scholl.

The difference graphs of S-units

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Abstract. Given a finite nonempty set of primes S, a graph $\mathcal{G}$ whose vertex set is the set of rational numbers and edge set is obtained by connecting vertices x and y if the prime divisors of both the numerator and denominator of $x-y$ are from S, is called the S-unit graph. Graphs of this type were first introduced by K. Győry in the 70's for addressing various Diophantine problems. We resolve two conjectures posed by I. Ruzsa concerning the possible sizes of induced nondegenerate cycles of $\mathcal{G}$, and also a problem of Ruzsa concerning the existence of subgraphs of $\mathcal{G}$ which are not induced subgraphs.

These results come from a joint work with A. Ćustić, L. Hajdu and R. Tijdeman.

On the counting function of irregular primes

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Abstract. A prime $p > 3$ is irregular if it divides the numerator of one of the Bernoulli numbers $B_2, \dots, B_{p-3}$. It is known that there are infinitely many irregular primes. In my talk, I will show that the number of irregular primes $p \le x$ is at least as large as $(1 + o(1)) \log\log x / \log\log\log x$ as $x \to \infty$. The proof uses sieves and linear forms in logarithms.

This is joint work with Amalia Pizarro from the University of Valparaiso, Chile and Carl Pomerance from Dartmouth College.

On the Mordell-Weil group of elliptic curves induced by families of Diophantine triples

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Abstract. The problem of the extendibility of Diophantine triples is closely connected with the Mordell-Weil group of the associated elliptic curve. In this talk Diophantine triples $\{k-1, k+1, c_l(k)\}$ will be examined, where

$$c_l(k) = \frac{(k + \sqrt{k^2-1})^{2l+1} + (k - \sqrt{k^2-1})^{2l+1} - 2k}{2(k^2-1)},$$

and it will be shown that the torsion group of the associated curves is $\mathbb{Z}/2\mathbb{Z} \times \mathbb{Z}/2\mathbb{Z}$ for $l = 3, 4$ and $l \equiv 1$ or $2 \pmod 4$. Additionally, the results about the rank of the curves obtained in this manner will be presented. Namely, rank is greater or equal to 2 for all $l \ge 2$, which is an improvement of previous results by Dujella, Pethő and Najman, where cases $k = 2$ and $l \le 3$ were considered.

Elliptic curves and Diophantine triples

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Abstract. A set $\{a_1, a_2, \ldots, a_m\}$ of m non-zero integers (rationals) is called *a (rational) Diophantine m-tuple* if $a_i \cdot a_j + 1$ is a perfect square for all $1 \leq i < j \leq m$. In this presentation, we consider elliptic curves of the form

$$y^2 = (ax+1)(bx+1)(cx+1),$$

where $\{a, b, c\}$ is a rational Diophantine triple. We say that this elliptic curve is induced by the Diophantine triple $\{a, b, c\}$.

By Mazur's theorem, there are at most four possibilities for the torsion group of such curves, namely, $\mathbb{Z}/2\mathbb{Z} \times \mathbb{Z}/2\mathbb{Z}$, $\mathbb{Z}/2\mathbb{Z} \times \mathbb{Z}/4\mathbb{Z}$, $\mathbb{Z}/2\mathbb{Z} \times \mathbb{Z}/6\mathbb{Z}$ and $\mathbb{Z}/2\mathbb{Z} \times \mathbb{Z}/8\mathbb{Z}$.

In this presentation, we study the rank of elliptic curves induced by Diophantine triples with torsion $\mathbb{Z}/2\mathbb{Z} \times \mathbb{Z}/4\mathbb{Z}$. The previous records for this torsion group were rank 8 over $\mathbb{Q}$ and rank ≥ 3 over $\mathbb{Q}(t)$.

We have found new examples of such curves over $\mathbb{Q}$ with rank 8 and one example with rank 9, and a parametric family of elliptic curves with torsion group $\mathbb{Z}/2\mathbb{Z} \times \mathbb{Z}/4\mathbb{Z}$ and with rank ≥ 4. Moveover, we will prove that its generic rank is equal to 4 and find the generators of the Mordell-Weil group by using a recent result of I. Gusić and P. Tadić.

This is a joint work with Andrej Dujella.

Common expansions in noninteger bases

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Abstract. In this talk we present results on the existence of simultaneous representations of real numbers in bases $1 < q_1 < \cdots < q_r, r \geq 2$ with the digit set $A = \{-m, \ldots, 0, \ldots, m\}$.

In the case $r = 2$ we prove among other results, that if $m = 1$ and $q_2 < 2$, then there is a continuum of sequences $(c_i) \in A^\infty$ satisfying

$$\sum_{i=1}^{\infty} \frac{c_i}{q_1^i} = \sum_{i=1}^{\infty} \frac{c_i}{q_2^i}.$$

On the other hand, if $m = 1$ and $q_2 \geq 2 + \sqrt{2}$, then only the trivial sequence $(c_i) = 0^\infty$ satisfies the former equality.

Let r be a positive integer. Then there exists a positive integer m and an interval I such that for all $z \in I$ there exist $1 < q_1 < \cdots < q_r$ and $(c_i) \in A^\infty$ with

$$z = \sum_{i=1}^{\infty} \frac{c_i}{q_1^i} = \cdots = \sum_{i=1}^{\infty} \frac{c_i}{q_r^i}.$$

The method of the proofs are completely different in the two cases. The proof of the first results are based on interval filling sequences, more precisely on a variant of a theorem of Kakeya, 1914. In contrast the general construction is based on CNS polynomials. This shows again a relation between CNS and β-expansions.

This is a joint work with V. Komornik.

Householder's approximants and continued fraction expansion of quadratic irrationals

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Abstract. Let α be a quadratic irrational. It is well known that the continued fraction expansion of α is periodic. We observe Householder's approximant of order $m-1$ for the equation $(x-\alpha)(x-\alpha') = 0$ and $x_0 = p_n/q_n$: $R_n^{(m)} = \frac{\alpha(p_n/q_n-\alpha')^m - \alpha'(p_n/q_n-\alpha)^m}{(p_n/q_n-\alpha')^m-(p_n/q_n-\alpha)^m}$. We say that $R_n^{(m)}$ is good approximant if $R_n^{(m)}$ is a convergent of α. When period begins with a_1, there is a good approximant at the end of the period, and when period is palindromic and has even length ℓ, there is a good approximant in the half of the period. So when $\ell \leq 2$, then every approximant is good, and then it holds $R_n^{(m)} = \frac{p_{m(n+1)-1}}{q_{m(n+1)-1}}$ for all $n \geq 0$. We prove that to be a good approximant is the palindromic and the periodic property. Further, we define the numbers $j^{(m)} = j^{(m)}(\alpha, n)$

by $R_n^{(m)} = \frac{p_{m(n+1)-1+2j}}{q_{m(n+1)-1+2j}}$ if $R_n^{(m)}$ is a good approximant. We prove that $|j^{(m)}|$ is unbounded by constructing an explicit family of quadratic irrationals, which involves the Fibonacci numbers.

On the decomposability of the linear combinations of Bernoulli polynomials

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Abstract. In the present talk we describe the complete decomposition of the linear combination of Bernoulli polynomials

$$R_n(x) = B_n(x) + cB_{n-2}(x),$$

where c is an arbitrary rational number.

The talk is based on a joint work with Csaba Rakaczki.

Rogers-Ramanujan type identities and quasiparticles in the principal picture of $\widehat{\mathfrak{sl}}_2$

Mirko Primc
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Abstract. J. Lepowsky and R. Wilson proved Rogers-Ramanujan identities by using representation theory of affine Lie algebra $\widehat{\mathfrak{sl}}_2$. The product sides of identites are explained by representation theory of Kac-Moody Lie algebras, and the combinatorial interpretation of sum sides is explained by representation theory of vertex operator algebras. In a joint work with S. Kožić we explain the sum sides of Rogers-Ramanujan type identities by using quasiparticles in the principal picture of $\widehat{\mathfrak{sl}}_2$.

$D(-1)$-triples of the form $\{1, b, c\}$ in the ring $\mathbb{Z}[\sqrt{-t}], t > 0$

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Abstract. We study $D(-1)$-triples of the form $\{1, b, c\}$ in the ring $\mathbb{Z}[\sqrt{-t}]$, $t > 0$, for positive integer b such that b is a prime, twice prime or twice prime squared. We prove that in those cases c has to be an integer. As a consequence of that result, in cases of $b = 26, 37$ or 50 we prove that $D(-1)$-triples of the form $\{1, b, c\}$ cannot be extended to a $D(-1)$-quadruple in the ring $\mathbb{Z}[\sqrt{-t}], t > 0$, except in cases of $t \in \{1, 4, 9, 25, 36, 49\}$. For those exceptional cases of t we show that there exist infinitely many $D(-1)$-quadruples of the form $\{1, b, -c, d\}, c, d > 0$ in $\mathbb{Z}[\sqrt{-t}]$.

On polynomial values of sums of Fibonacci numbers

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Abstract. Let $p(x) \in \mathbb{Z}[x]$ with $p(\mathbb{N}) \subset \mathbb{N}$ be a fixed polynomial. The aim of the talk is to present the following result: There is $C = C(p) > 0$ and $N_0 = N_0(p) > 0$, both effective, such that for all $N \geq N_0$ there is a positive integer n that is the sum of exactly N Fibonacci numbers and $p(x)$ is the sum of at most $C(p)$ Fibonacci numbers. For example, for $p(x) = x^h$ one can take $N_0 = 8h + 1$ and $C = (2h + 1)(8h + 2)$. We put the result in the context of some problems and results due to Gelfond (1967/68), Stolarsky (1978) and Lindström (1997).

Some connections between non-commutative harmonic analysis and the theory of automorphic forms

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Abstract. Some very important problems of modern theory of automorphic forms are typical problems of non-commutative harmonic analysis (in a broad sense). From the other side, in building harmonic analysis on reductive groups, automorphic forms are very useful. They are very rich source of relevant ideas, as well as crucial representations and concepts. In the talk we shall review some of these connections.

Injectivity of the specialization homomorphism of elliptic curves

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Abstract. Let E be a nonconstant elliptic curve over $\mathbb{Q}(t)$ with at least one nontrivial $\mathbb{Q}(t)$-rational 2-torsion point. By the Silverman specialization theorem the specialization homomorphism $t \mapsto t_0$ is injective for all but finitely many $t_0 \in \mathbb{Q}$. We describe a method for finding $t_0 \in \mathbb{Q}$ for which the corresponding specialization homomorphism is injective. The method can be directly extended to elliptic curves over $K(t)$ for a number field K of class number 1, and in principal for arbitrary number field K. This is an improvement of a result from one of our former preprints on this topic. We show how this method can be used to calculate the rank of elliptic curves over $\mathbb{Q}(t)$ of the form as above, by observing a chosen specialized curve.

This is a joint work with I. Gusić.

Polynomial Diophantine equations

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Abstract. In this lecture polynomial Diophantine equations of the separated variables type $f(x) = g(y)$ are considered. The history of such equations, connections to Ritts theorems and recent applications are surveyed. We mainly focus on applications of Siegels theorem to obtain finiteness of the number of solutions. Effective results are discussed for very specials cases. Finally, we report an decomposibility results for polynomials, including joint work with A. Dujella and I. Gusić as well as some recent results of D. Kreso and M. Zieve.

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