Left Invertibility of Formal Hamiltonian Operators

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Definition

Let X be a complex infinite dimensional Hilbert space. The formal Hamiltonian operator is given by the closed densely defined block operator matrix

$$H = \begin{pmatrix} A & B \\ C & -A^* \end{pmatrix} : \mathcal{D}(H) \subset X \oplus X \to X \oplus X,$$

where A is a densely defined closed operator, and B and C are symmetric operators.

- If B and C are self-adjoint, then H is Hamiltonian which naturally arise from linear infinite dimensional Hamiltonian systems. Due to the requirement in applications, one is also interested in the case of symmetric operators B and C.
- The formal Hamiltonian operator is of symplectic symmetry, i.e., $JH \subset (JH)^*$ with $J = \begin{pmatrix} 0 & I \\ -I & 0 \end{pmatrix}$.

Completion Problem

Definition

Partial operator matrix is operator matrix whose entries are specified only on a subset of its positions, while a completion of a partial operator matrix is the operator matrix resulting from filling in its unspecified entries.

We are devoted to find the conditions on A and C such that the partial formal Hamiltonian operator

$$\begin{pmatrix} A & ? \\ C & -A^* \end{pmatrix}$$

has a left invertible completion.

- The operator matrix completion problem was shown to be very useful in operator theory, numerical analysis, optimal control, system theory and engineering problems.
- In operator matrix completion problems, one concerns the conditions under which a partial operator matrix has completions with some given properties (left invertibility etc.).