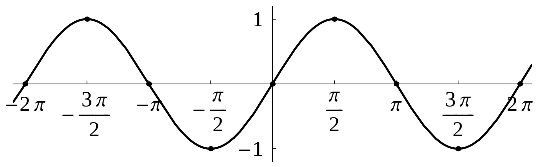
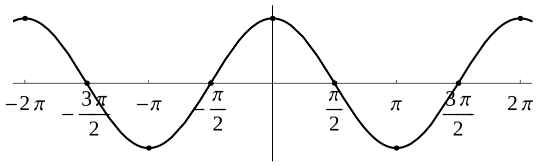
Trigonometrijske funkcije

- $f(x) = \sin x$



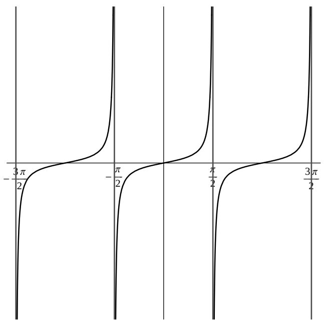
$\mathcal{D}_f = \mathbb{R}$
$\mathcal{R}_f = [-1, 1]$

- $f(x) = \cos x$



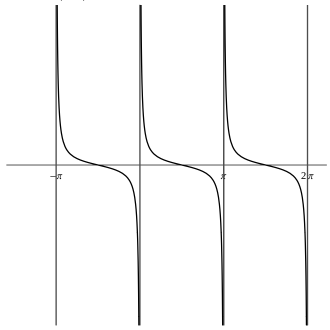
$\mathcal{D}_f = \mathbb{R}$
$\mathcal{R}_f = [-1, 1]$

- $f(x) = \operatorname{tg} x$



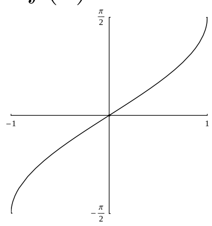
$\mathcal{D}_f = \mathbb{R} \setminus \{\frac{\pi}{2} + k\pi \colon k \in \mathbb{Z}\}$
$\mathcal{R}_f = \mathbb{R}$

- $f(x) = \operatorname{ctg} x$



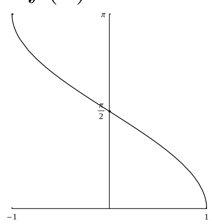
$\mathcal{D}_f = \mathbb{R} \setminus \{k\pi \colon k \in \mathbb{Z}\}$
$\mathcal{R}_f = \mathbb{R}$

- $f(x) = \arcsin x$



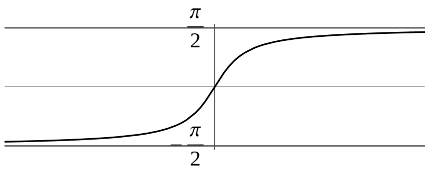
$\mathcal{D}_f = [-1, 1]$
$\mathcal{R}_f = [-\frac{\pi}{2}, \frac{\pi}{2}]$

- $f(x) = \arccos x$



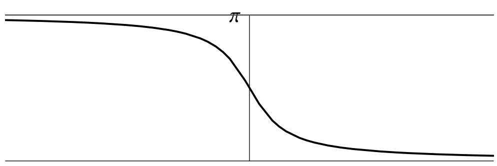
$\mathcal{D}_f = [-1, 1]$
$\mathcal{R}_f = [0, \pi]$

- $f(x) = \operatorname{arctg} x$



$\mathcal{D}_f = \mathbb{R}$
$\mathcal{R}_f = \langle -\frac{\pi}{2}, \frac{\pi}{2} \rangle$

- $f(x) = \operatorname{arcctg} x$



$\mathcal{D}_f = \mathbb{R}$
$\mathcal{R}_f = \langle -0, \pi \rangle$

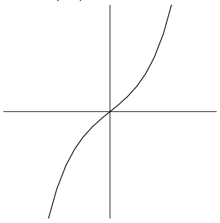
$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$
$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$
$\sin x \pm \sin y = 2 \sin \frac{x \pm y}{2} \cos \frac{x \mp y}{2}$
$\cos x + \cos y = 2 \cos \frac{x+y}{2} \cos \frac{x-y}{2}$
$\cos x - \cos y = -2 \sin \frac{x+y}{2} \sin \frac{x-y}{2}$

$\operatorname{tg}(x \pm y) = \frac{\operatorname{tg} x \pm \operatorname{tg} y}{1 \mp \operatorname{tg} x \operatorname{tg} y}$
$\operatorname{ctg}(x \pm y) = \frac{\operatorname{ctg} x \operatorname{ctg} y \mp 1}{\operatorname{ctg} y \pm \operatorname{ctg} x}$
$\sin x \sin x = \frac{\cos(x-y) - \cos(x+y)}{2}$
$\sin x \cos x = \frac{\sin(x+y) + \sin(x-y)}{2}$
$\cos x \cos x = \frac{\cos(x-y) + \cos(x+y)}{2}$

$\sin 2x = 2 \sin x \cos x$
$\cos 2x = \cos^2 x - \sin^2 x$
$\sin^2 x + \cos^2 x = 1$

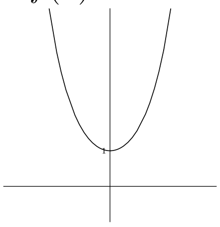
Hiperbolne funkcije

- $f(x) = \operatorname{sh} x$



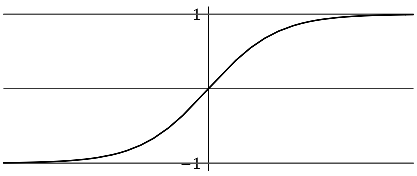
$\mathcal{D}_f = \mathbb{R}$
$\mathcal{R}_f = \mathbb{R}$

- $f(x) = \operatorname{ch} x$



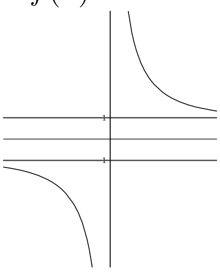
$\mathcal{D}_f = \mathbb{R}$
$\mathcal{R}_f = [1, +\infty\rangle$

- $f(x) = \operatorname{th} x$



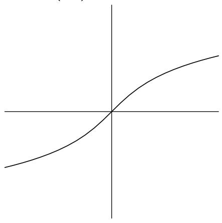
$\mathcal{D}_f = \mathbb{R}$
$\mathcal{R}_f = \langle -1, 1\rangle$

- $f(x) = \operatorname{cth} x$



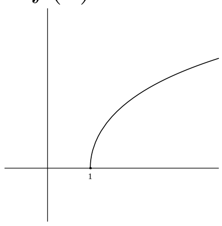
$\mathcal{D}_f = \mathbb{R} \setminus \{0\}$
$\mathcal{R}_f = \langle -\infty, -1\rangle \cup \langle 1, +\infty\rangle$

- $f(x) = \operatorname{Arsh} x$



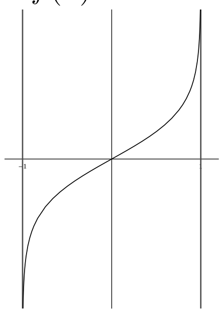
$\mathcal{D}_f = \mathbb{R}$
$\mathcal{R}_f = \mathbb{R}\rangle$

- $f(x) = \operatorname{Arch} x$



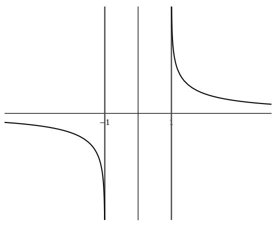
$\mathcal{D}_f = [1, +\infty\rangle$
$\mathcal{R}_f = [0, +\infty\rangle$

- $f(x) = \operatorname{Arth} x$



$\mathcal{D}_f = \langle -1, 1\rangle$
$\mathcal{R}_f = \mathbb{R}$

- $f(x) = \operatorname{Arcth} x$



$\mathcal{D}_f = \langle -\infty, -1\rangle \cup \langle 1, +\infty\rangle$
$\mathcal{R}_f = \mathbb{R} \setminus \{0\}$

$\operatorname{sh}(x \pm y) = \operatorname{sh} x \operatorname{ch} y \pm \operatorname{ch} x \operatorname{sh} y$
$\operatorname{ch}(x \pm y) = \operatorname{ch} x \operatorname{ch} y \pm \operatorname{sh} x \operatorname{sh} y$

$\operatorname{th}(x \pm y) = \frac{\operatorname{th} x \pm \operatorname{th} y}{1 \pm \operatorname{th} x \operatorname{th} y}$
$\operatorname{cth}(x \pm y) = \frac{\operatorname{cth} x \operatorname{cth} y \pm 1}{\operatorname{cth} y \pm \operatorname{cth} x}$

$\operatorname{sh} 2x = 2 \operatorname{sh} x \operatorname{ch} x$
$\operatorname{ch} 2x = \operatorname{ch}^2 x + \operatorname{sh}^2 x$
$\operatorname{ch}^2 x - \operatorname{sh}^2 x = 1$

Tablica limesa

$$\lim_{x\to 0}\frac{\sin x}{x}=1 \qquad \lim_{x\to 0}\frac{\operatorname{sh} x}{x}=1$$

$$\lim_{x\to 0}\frac{1-\cos x}{x^2}=\frac{1}{2} \qquad \lim_{x\to 0}\frac{\operatorname{ch} x-1}{x^2}=\frac{1}{2}$$

$$\lim_{x\to 0}(1+x)^{\frac{1}{x}}=e \qquad \lim_{x\to\pm\infty}\left(1+\frac{1}{x}\right)^x=e$$

$$\lim_{x\to 0}\frac{e^x-1}{x}=1 \qquad \lim_{x\to 0}\frac{\ln(1+x)}{x}=1$$

$$\lim_{x\to 0}\frac{a^x-1}{x}=\ln a \qquad (a>0) \qquad \lim_{x\to 0}\frac{(1+x)^a-1}{x}=a \qquad (a\in\mathbb{R})$$

$$\lim_{x\to+\infty}\frac{x^p}{a^x}=0 \qquad (p\in\mathbb{R},\ a>1)$$

$$\lim_{x\to\pm\infty}\frac{a_m x^m+a_{m-1}x^{m-1}+\cdots+a_1x+a_0}{b_n x^n+b_{n-1}x^{n-1}+\cdots+b_1x+b_0}=\begin{cases}\frac{a_m}{b_n} & \text{kada je } m=n\\ 0 & \text{kada je } m<n\\ \pm\infty & \text{kada je } m>n\end{cases}$$

$(m,n\in\mathbb{N}_0,\quad a_0,\dots,a_m,b_0,\dots,b_n\in\mathbb{R},\quad a_m,b_n\neq 0)$

Limesi oblika $\lim\limits_{x\to c}\varphi(x)^{\psi(x)}$

Neka je $\lim\limits_{x\to c}\varphi(x)=A$, $0<A\leq+\infty$, $\lim\limits_{x\to c}\psi(x)=B$, $-\infty\leq B\leq+\infty$, pri čemu je $-\infty\leq c\leq+\infty$.

1° Ako je $B\in\mathbb{R}$, onda vrijedi

$$\lim_{x\to c}\varphi(x)^{\psi(x)}=A^B$$

2° Ako je $A\neq 1$, $B=\pm\infty$, onda vrijedi

$$\lim_{x\to c}\varphi(x)^{\psi(x)}=\begin{cases}+\infty & \text{kada je } A<1,\ B=-\infty\\ 0 & \text{kada je } A<1,\ B=+\infty\\ 0 & \text{kada je } A>1,\ B=-\infty\\ +\infty & \text{kada je } A>1,\ B=+\infty\end{cases}$$

3° Ako je $A=1$, $B=\pm\infty$, onda se limes računa po formuli

$$\lim_{x\to c}\varphi(x)^{\psi(x)}=e^{\lim_{x\to c}(\varphi(x)-1)\psi(x)}$$

Tablica derivacija

$c' = 0 \qquad (c \in \mathbb{R} \text{ konstanta})$

$x' = 1$

$(x^n)' = nx^{n-1} \qquad (n \in \mathbb{Z})$

$(x^a)' = ax^{a-1} \qquad (a \in \mathbb{R},\ x > 0)$

$(\sqrt{x})' = \dfrac{1}{2\sqrt{x}} \qquad (x > 0)$

$(\sin x)' = \cos x$

$(\cos x)' = -\sin x$

$(\operatorname{tg} x)' = \dfrac{1}{\cos^2 x}$

$(\operatorname{ctg} x)' = -\dfrac{1}{\sin^2 x}$

$(\arcsin x)' = \dfrac{1}{\sqrt{1-x^2}} \qquad (|x| < 1)$

$(\arccos x)' = -\dfrac{1}{\sqrt{1-x^2}} \qquad (|x| < 1)$

$(\operatorname{arctg} x)' = \dfrac{1}{1+x^2}$

$(\operatorname{arcctg} x)' = -\dfrac{1}{1+x^2}$

$(a^x)' = a^x \ln a \qquad (a > 0)$

$(e^x)' = e^x$

$(\log_a x)' = \dfrac{1}{x \ln a} \qquad (a > 0,\ a \neq 1,\ x > 0)$

$(\ln x)' = \dfrac{1}{x} \qquad (x > 0)$

$(\operatorname{sh} x)' = \operatorname{ch} x$

$(\operatorname{ch} x)' = \operatorname{sh} x$

$(\operatorname{th} x)' = \dfrac{1}{\operatorname{ch}^2 x}$

$(\operatorname{cth} x)' = -\dfrac{1}{\operatorname{sh}^2 x}$

$(\operatorname{Arsh} x)' = \dfrac{1}{\sqrt{1+x^2}}$

$(\operatorname{Arch} x)' = \dfrac{1}{\sqrt{x^2-1}} \qquad (x > 1)$

$(\operatorname{Arth} x)' = \dfrac{1}{1-x^2} \qquad (|x| < 1)$

$(\operatorname{Arcth} x)' = \dfrac{1}{1-x^2} \qquad (|x| > 1)$

Pravila deriviranja

$(u(x) \pm v(x))' = u'(x) \pm v'(x)$

$(c \cdot u(x))' = c \cdot u'(x)$

$(u(x) \cdot v(x))' = u'(x)v(x) + u(x)v'(x)$

$$\left(\frac{u(x)}{v(x)}\right)' = \frac{u'(x)v(x) - u(x)v'(x)}{v(x)^2}$$

$$\left(\frac{1}{v(x)}\right)' = -\frac{v'(x)}{v(x)^2}$$

$$\Big(f(g(x))\Big)' = f'(g(x)) \cdot g'(x)$$

Derivacije višeg reda

$(a^x)^{(n)} = a^x \ln^n a \qquad (a > 0)$

$(\sin x)^{(n)} = \sin\left(x + \frac{n\pi}{2}\right)$

$(\cos x)^{(n)} = \cos\left(x + \frac{n\pi}{2}\right)$

$$(\operatorname{sh} x)^{(n)} = \begin{cases} \operatorname{sh} x, & n \ \text{paran} \\ \operatorname{ch} x, & n \ \text{neparan} \end{cases}$$

$$(\operatorname{ch} x)^{(n)} = \begin{cases} \operatorname{ch} x, & n \ \text{paran} \\ \operatorname{sh} x, & n \ \text{neparan} \end{cases}$$

$(x^m)^{(n)} = m(m-1)\cdots(m-n+1)x^{m-n} \quad (m \in \mathbb{Z})$

$(u \cdot v)^{(n)}(x) = \sum_{k=0}^{n} \binom{n}{k} u^{(k)}(x) \cdot v^{(n-k)}(x)$

Tablica integrala

$$\int dx = x + C$$

$$\int x^a\,dx = \frac{x^{a+1}}{a+1} + C \qquad (a \neq -1)$$

$$\int \frac{dx}{x} = \ln|x| + C$$

$$\int e^x\,dx = e^x + C$$

$$\int a^x\,dx = \frac{a^x}{\ln a} + C \qquad (a > 0,\ a \neq 1)$$

$$\int \cos x\,dx = \sin x + C$$

$$\int \sin x\,dx = -\cos x + C$$

$$\int \frac{dx}{\cos^2 x} = \operatorname{tg} x + C$$

$$\int \frac{dx}{\sin^2 x} = -\operatorname{ctg} x + C$$

$$\int \operatorname{ch} x\,dx = \operatorname{sh} x + C$$

$$\int \operatorname{sh} x\,dx = \operatorname{ch} x + C$$

$$\int \frac{dx}{\operatorname{ch}^2 x} = \operatorname{th} x + C$$

$$\int \frac{dx}{\operatorname{sh}^2 x} = -\operatorname{cth} x + C$$

$$\int \frac{dx}{\sqrt{1-x^2}} = \arcsin x + C$$

$$\int \frac{dx}{1+x^2} = \operatorname{arctg} x + C$$

$$\int \frac{dx}{\sqrt{1+x^2}} = \operatorname{Arsh} x + C = \ln\left(x + \sqrt{1+x^2}\right) + C$$

$$\int \frac{dx}{\sqrt{x^2-1}} = \operatorname{Arch} x + C = \ln\left|x + \sqrt{x^2-1}\right| + C$$

$$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C \qquad (a > 0)$$

$$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\operatorname{arctg}\frac{x}{a} + C \qquad (a > 0)$$

$$\int \frac{dx}{\sqrt{a^2+x^2}} = \ln\left(x + \sqrt{a^2+x^2}\right) + C \qquad (a > 0)$$

$$\int \frac{dx}{\sqrt{x^2-a^2}} = \ln\left|x + \sqrt{x^2-a^2}\right| + C \qquad (a > 0)$$

$$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left|\frac{x+a}{x-a}\right| + C \qquad (a > 0)$$

Taylorovi redovi

1. $e^x = 1 + x + \dfrac{x^2}{2!} + \dfrac{x^3}{3!} + \dots \; = \displaystyle\sum_{n=0}^{\infty} \frac{x^n}{n!},\ x \in \mathbb{R}$

2. $\cos x = 1 - \dfrac{x^2}{2!} + \dfrac{x^4}{4!} - \dfrac{x^6}{6!} + \dots \; = \displaystyle\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!},\ x \in \mathbb{R}$

3. $\sin x = x - \dfrac{x^3}{3!} + \dfrac{x^5}{5!} - \dfrac{x^7}{7!} + \dots \; = \displaystyle\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!},\ x \in \mathbb{R}$

4. $\dfrac{1}{1-x} = 1 + x + x^2 + x^3 + \dots = \displaystyle\sum_{n=0}^{\infty} x^n,\ |x| < 1$

5. $(1+x)^\alpha = 1 + \dbinom{\alpha}{1} x + \dbinom{\alpha}{2} x^2 + \dots = \displaystyle\sum_{n=0}^{\infty} \binom{\alpha}{n} x^n,\ |x| < 1,$

   pri čemu je $\dbinom{\alpha}{n} = \dfrac{\alpha(\alpha-1)\cdots(\alpha-n+1)}{n!},\ \dbinom{\alpha}{0} = 1$

6. $\sqrt{1+x} = (1+x)^{\frac{1}{2}} = 1 + \displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n-1}(2n-3)!!}{2^n\, n!}\, x^n,\ |x| < 1,$

   jer je $\dbinom{\frac{1}{2}}{n} = \dfrac{(-1)^{n-1}(2n-3)!!}{2^n\, n!}$

7. $\dfrac{1}{\sqrt{1+x}} = (1+x)^{-\frac{1}{2}} = 1 + \displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n(2n-1)!!}{2^n n!}\, x^n,\ |x| < 1,$

   jer je $\dbinom{-\frac{1}{2}}{n} = \dfrac{(-1)^n(2n-1)!!}{2^n n!}$

8. Ako je P polinom stupnja m, onda je

   $$P(x) = P(0) + \frac{P'(0)}{1!} x + \frac{P''(0)}{2!} x^2 + \dots = \sum_{n=0}^{m} \frac{P^{(n)}(0)}{n!}\, x^n,\ |x| < 1$$

9. $\ln(1+x) = \dfrac{x}{1} - \dfrac{x^2}{2} + \dfrac{x^3}{3} - \dfrac{x^4}{4} + \dots = \displaystyle\sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^n}{n},\ |x| < 1$

10. $\operatorname{arctg} x = \dfrac{x}{1} - \dfrac{x^3}{3} + \dfrac{x^5}{5} - \dfrac{x^7}{7} + \dots = \displaystyle\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1},\ |x| < 1$

11. $\arcsin x = \displaystyle\sum_{n=0}^{\infty} \frac{(2n-1)!!}{2^n\, n!} \cdot \frac{x^{2n+1}}{2n+1},\ |x| < 1$